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AERODYNAMICAL AND MECHANICAL FACTORS INFLUENCING THE TERMINAL VELOCITY OF A DOWNHILL SKIER

by

Hans Gros

(C)

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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FALL 1978

THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled AERODYNAMICAL AND MECHANICAL FACTORS INFLUENCING THE VELOCITY OF A DOWNHILL SKIER submitted by Hans Josef Gros in partial fulfilment of the requirements for the degree of Master of Science.

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Supervisor

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DATED: October 7, 1978

ABSTRACT

The purpose of this study was to quantify the influence of selected variables on the terminal velocity of a downhill skier. A computermodel representing a skier in a uniform position on a uniform slope was constructed. Expressions quantifying the importance of the variables: mass of the skier-ski system, angle of the slope, coefficient of friction, planform area, and coefficient of drag were derived from the basic equation for terminal velocity. A field study was administered to see whether the results concerning the influence of added mass on the terminal velocity could be reproduced.

The fact that a mass increment theoretically produces a higher terminal velocity has been stated in the literature. The importance of aerodynamic considerations is widely accepted. The results of this study agree with the literature. Furthermore, the presented findings allow quantification of the impact of selected variables and prediction of terminal velocity for specified cases. The results of the field experiments confirm the validity of the computer model. The findings are summarized in tables that allow easy and fast estimation of the per-cent change in terminal velocity caused by a defined change of one of the variables. This study indicates that the feasibility of improving the time of a downhill run by adding mass to the skier-ski system has to be investigated under racing conditions to see whether a net gain in time materializes.

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Chapter I

Statement of the Problem

Introduction

The coach, trying to assess the performance of a downhill skier, usually has to rely on the total time for a run (as compared to other racers), intermediate times, direct observation of one section of the race and video or film documentation.

It is usually impossible to say what exactly caused a run to be faster than other runs because of the complexity of the event (i.e. the large number of influencing variables) and the fact that slight changes (e.g. in body position) are of considerable importance. It is also virtually impossible to determine how important different factors are for the total performance. Pure empirical research in this case seems to be of limited value because only results and not reasons are demonstrated.

The key to an insight in the event is thus a method that allows isolation of important variables. The question is: (a) What is important, (b) How important is it and (c) Why is it important. Answers to these questions can be brought closer to a solution by determining the influence of one single factor on the performance. In the case of downhill skiing intervening variables make it nearly impossible to look at one variable at a time in an experimental environment. Thus both the theoretical and the empirical approach must be used as methods of providing the coach with tools to estimate the impact of selected variables on performance. This study concentrates

on factors that determine terminal velocity (top speed) of the skier and on suggestions on how to increase it. An increase in terminal velocity should bring a potential improvement of the overall performance.

Purpose of the Study

The purpose of this study was to quantify the influence of selected variables on terminal velocity in downhill skiing. In particular these variables include: (a) mass of the skier-ski system, (b) angle of the slope, (c) coefficient of friction between ski and snow, (d) planform area of the skier and (e) coefficient of drag.

Definition of Terms

A — Angle of the slope in degrees.

A_c — Critical angle for impending motion. The smallest angle of the slope at which a skier will start sliding for a given coefficient of friction.

a — The acceleration of a given mass under the influence of a force.

CD — The coefficient of drag of the skier.

d — The density of the air.

F_a — The net accelerating force obtained as the difference between the force component parallel to the slope (F_x) and the friction force (F_f).

F_d — The drag force defined as the product of the square of the velocity, the planform area (PA), the coefficient of drag (CD) and 1/2 the density of the air (d).

F_f The friction force is the product of the normal component of force (F_n) and the coefficient of friction (u).

F_n The normal component of the force.

F_x The force component parallel to the slope.

g The acceleration due to gravity.

m The mass of the skier-ski system.

PA The planform (cross-sectional) area of the skier.

s A defined displacement of the skier.

u The coefficient of friction between ski and snow.

V The velocity of the skier.

V_s The velocity of the skier after a defined displacement (s).

V_t The terminal velocity of the skier under defined conditions.

Chapter II

Review of Literature

The coefficient of friction between the ski and the snow surface has been investigated by several researchers. The findings suggest that the coefficients of static and kinetic frictions are subject to substantial changes caused by factors such as: snow conditions; snow and air temperature; as well as smoothness, material, and preparation of the ski sole. (OUTWATER 1970) (SHIMBO, 1971). Therefore, it has to be assumed, that the results reported hold true only for specific situations. KINOSITA (1971) determined the coefficient of kinetic friction to be .05. FUKUOKA (1971) p.48) reports values of .1 to .05 and .01 to .05 for the coefficients of static and kinetic friction respectively. SHIMBO (1971) tried to eliminate intervening variables by measuring the friction of a ring sliding on a snow-filled pan in a controlled laboratory setting. He found the coefficient of friction to be independent of running speed. Range tested: .1 to 7.2 m/s. Loading the sliding ring with different weights produced similar coefficients of friction. A range of 21 to 66 g/cm^2 was tested. The pressure exerted by a skier is 20 to 25 g/cm^2. SHIMBO tested a variety of ski base materials. The most frequently used high density polyethylenes had a coefficient of kinetic friction of .02 to .09.

The importance of the mass of the skier-ski system for the terminal velocity has been recognized by several authors. SCHOECK (1963) states that the heavier skiers have an advantage because the velocity gain caused by the greater mass will generally overcompensate

the loss caused by the increased planform area. FUKUOKA (1971 p.24) supports this statement. In a study basically concerned with aerodynamic factors related to downhill skiing TEMPLIN (1968) mentions the possibility of adding extra mass to the skier-ski system to produce higher terminal velocities. To our knowledge no attempt has been made to quantify the influence of added mass on the terminal velocity.

The important role of aerodynamic factors such as planform area or coefficient of drag for downhill skiing is widely recognized. A number of windtunnel experiments were performed to find the optimum body position and the best racing suit for downhill skiers. (RAINE, 1970), (FUETTERER, 1960). TEMPLIN (1968) derived equations to estimate terminal velocity. He also solved the time and displacement equations for a skier in a uniform position on a slope with known rise and run. THOMANN (1976) calculated the time loss caused by an increase of planform area for a defined time interval. He also reports values of .3 m^2 (deep crouch) and 1.2 m^2 (upright stance) for the planform area. In accordance with SCHOECK (1963) THOMANN states that the drag reduction achieved by the so-called 'egg position' is caused by a decrease of planform area rather than a decrease of the coefficient of drag. GORLIN et.al. (1972) tested subjects in a windtunnel with relative velocities between 10 and 45 m/s. The coefficient of drag was found to be virtually independent of the body position of the skier.

In summary it has to be mentioned that the available literature pertaining to the specific problems of this study is rather limited. This reflects the fact that biomechanical research does not yet play a major role in this sporting event.

Chapter III

Methods and Procedure

The following chapter describes the methods used to construct the mechanical model and the procedure followed for the field experiment as well as the limitations of this study.

The Mechanical Model

A mechanical model was constructed with the following relevant variables for the overall performance considered:

(a) The angle of the slope.

(b) The mass of the skier-ski system.

(c) The coefficient of friction between ski and snow.

(d) The planform area of the skier.

(e) The coefficient of drag of the skier.

Several programs were run on the computer to compute terminal velocity from the derived equations. It was possible to determine the influence of each variable on the terminal velocity through the use of the computermodel. The assumed input values were validated by comparison with results of related experimental research. A Hewlett & Packard 9825A Computer was used for the data processing. All programs were written in Hewlett Packard Language (HPL).

The Field Experiment

One expert male skier skied down the fall-line of a natural hill assuming two different positions: standing and crouching

(racing position). The subject performed four runs in each position. In four of the eight runs the skier carried a weightbelt (8 kg). The subject was asked to keep his body position constant and to reproduce it from run to run.

The angle of the slope was measured with a hubsometer and checked using a film recording. The run-up length to the measuring section was 45 meters. All runs were filmed with a Photo-Sonics 1 PL 16 mm camera equipped with an electronic timing light generator set at 10 Hz. The camera was operated at 50 frames per second with a shutter angle of 120°, exposure time of 1/150 second and the f-stop set at 2.5. All shots were taken with Kodachrome 25 ASA film. The camera was placed perpendicular to the measuring section at a distance of 65 meters.

The datafilm was analyzed using a HP 9825A Computer hardwired to a HP 9864A Digitizer and a Bendix digitizing board. The system allows determination of cartesian coordinate points accurate to .0056 cm. The total accuracy of the film analysis was dependent on the resolution of the film and the skill of the operator. A comparison of repeated measures did not produce relative errors for the determined velocities greater than ± .15 m/s. The real size - image size ratio was 12.35 to 1. A TRIAD VR-100 film analyzer was used for projection onto the Bendix digitizing board.

The computerized programs rotated the internal X and Y axis of the platen to match the angle of the slope. Internal body angles were computed using the cosine law. Furthermore the velocity of the skier was calculated. In cases where the body position changed throughout the run the velocity of the center of mass

was determined, otherwise a surface mark was used. The programs also accommodated and corrected for slight variations of the frame rate based on the timing marks.

Limitations of the Study

The 'in vitro' method used has a number of inherent limitations. As only mathematically quantifiable variables can be considered a certain degree of simplification cannot be avoided. For some of the variables a value based on the findings of the related research has to be assumed. The mechanical model only considers the case of a skier skiing down the fall-line of a uniform slope in a uniform position.

Although the method of analysis used for the field experiment allowed an accurate determination of velocities by reducing the time and displacement error, the experiment must be seen in the context of the findings of the theoretical approach. A true validation of the model would require a very definite control of the main variables and a larger number of trial runs. This could not be achieved because of technical difficulties. Furthermore, the available run-up length of 45 meters was not sufficient to produce terminal velocity. The experiment should be viewed as an indication of the tendencies observed with the mechanical model.

Chapter IV

Results and Discussion

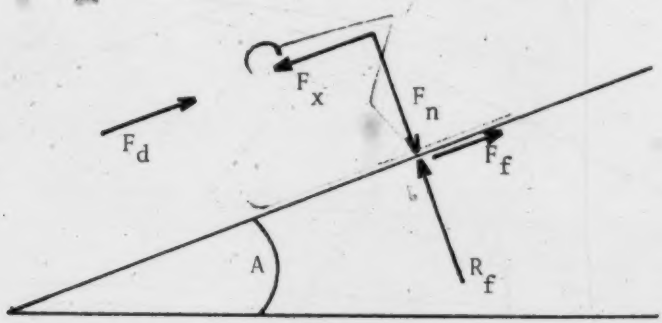
This chapter reports the findings obtained from the theoretical approach (namely the mechanical model) and the field experiment administered.

The Basic Mechanical Model

Under the defined conditions:

1. Uniform slope,
2. Uniform position.

The forces acting on the skier - ski system are as follows:



Where:

F_x: Force component parallel to the slope.

F_n: The normal force.

F_f: The friction force.

F_d: The drag force.

A: The angle of the slope.

R_f: The reaction force.

The Motion of the Skier disregarding Air Resistance

(a) For a given mass of the skier and a given angle of the slope, the force acting parallel to the slope tends to accelerate the skier and is defined by the relation:

$$F_x = m\ g\ \sin A$$

The force normal to the slope is the product of the mass the acceleration due to gravity and the cosine of the angle:

$$F_n = m\ g\ \cos A$$

The system is in uniformly accelerated motion.

(b) Knowing the normal force and assuming the coefficient of friction between ski and snow being independent of Velocity and F_n; the friction force (F_f) can be obtained as:

$$F_f = F_n\ u$$

The net accelerating force can thus be defined as:

$$m\ g\ \sin A - m\ g\ \cos A\ u \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

The equations for uniformly accelerated motion still apply:

$$V = at \text{ and } s = \frac{1}{2}\ at^2$$

where: $a = \dfrac{F_x - F_f}{\text{Mass}}$

It is obvious that the mass has no influence on the velocity. V is determined by the angle and the magnitude of the coefficient of friction. The friction force reduces the net accelerating force. The resulting loss in velocity is shown in Figure 1.

The expression derived from the net accelerating force is only true if $\sin A > \cos A\ u$.

The Motion of the Skier considering Air Resistance

The total drag or air resistance can be calculated using the standard formula:

$$\text{DRAG} = \frac{d}{2} V^2 \text{ (PA) (CD)}$$

Where:

d : density of the air.

V : velocity of the moving object.

PA: the planform area of the object.

CD: the coefficient of drag of the object.

Drag increases as the parabolic function of velocity. Thus the skier will accelerate until the drag force equals the net accelerating force. As the rate of change of velocity decreases, the velocity curve levels off. When dv/dt = 0 the skier has attained terminal velocity. The force accelerating the skier can be written as:

$$m\ g \sin A - m\ g \cos A\ u - \frac{d}{2} V^2 \text{ (PA) (CD)} = m\ a$$

Terminal velocity is calculated by equating the expressions for drag and the net accelerating force:

$$\frac{d}{2} V_t^2 \text{ (PA) (CD)} = m\ g \sin A - m\ g \cos A\ u \quad \ldots\ldots\ldots \quad (2)$$

solving for V_t we obtain:

$$V_t = \sqrt{\frac{2\ m\ g\ (\sin A - \cos A\ u)}{d\ \text{(PA) (CD)}}} \quad \ldots\ldots\ldots\ldots\ldots \quad (3)$$

From this equation it becomes obvious that the terminal velocity is influenced by: mass, angle of slope, planform area, and the coefficients of drag and friction. The density of the air and the acceleration due to gravity are assumed constant.

The Velocity of the Skier as a Function of the Displacement

The velocity of a skier after an arbitrary displacement (s) under defined conditions can be obtained as:

$$V^2 = \frac{2mg\ (\sin A - \cos A\ u)}{d\ (PA)\ (CD)} \times \left(1 - e^{\frac{-d\ (PA)\ (CD)\ s}{m}}\right) \qquad \text{(Pierce, 1956)}$$

In the case of terminal velocity the displacement the following end-conditions apply:

$s \rightarrow \infty$ thus $e^{-\infty} \rightarrow 0$

Hence, the above given equation reduces to the expression for terminal velocity (3) derived in the preceeding section.

The velocity as a function of the displacement (s) can thus be written as:

$$V_s = V_t \sqrt{\left(1 - e^{\frac{-d\ (PA)\ (CD)\ s}{m}}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (4)$$

Where: V_s = the velocity of the skier after displacement s.

The Influence of Selected Variables on Terminal Velocity

The following part of this thesis deals only with terminal velocity because:

(a) Under normal racing conditions one can expect only minute differences between the equipment used and the level of proficiency of the competitors. Figures (2) and (3) give examples of velocity curves with slightly modified input values for the coefficient of friction and the coefficient of drag. It becomes obvious that the difference in terminal velocities reflects the difference in veloci-

ties after a sufficient run-up. For smaller displacements the difference in actual velocities decreases. Figure (4) illustrates an extreme case where a skier has unfavourable friction values and low values for planform area and the coefficient of drag whereas the second skier is in an aerodynamically unfavourable body position but has skis with low friction. Whenever such extreme differences occur terminal velocity cannot be used to predict differences in submaximal velocities. However, this is not likely to occur in high level competition.

(b) Considering only terminal velocity permits the quantification of the impact of a single variable on terminal velocity under arbitrary conditions.

The Angle of the Slope

From the basic equation of motion:

$$\frac{d}{2} V^2 \text{ PA CD} = m\ g \sin A - m\ g \cos A\ u \quad \ldots\ldots\ldots\ldots (2)$$

we see that the magnitude of the angle influences both the accelerating and the decelerating forces:

$$F_x = m\ g \sin A$$

$$F_f = m\ g \cos A\ u$$

For increasing angles, F_x becomes bigger as a function of the sinus of the angle.

Example: While A goes from 0° to 40°, sin A increases from 0 to .643.

Increasing the angle of the slope also reduces the normal force and

thereby the friction force.

Example: While increasing A from 0° to 40°, the cos A goes from 1.0 to .766.

This indicates that at an angle of 50° the influence of the friction force is reduced to 76.6% of its original value. (Compare Figure 5).

An increase in the angle of the slope thus has a positive effect on the terminal velocity because it increases the accelerating force while decreasing the decelerating force. For a given coefficient of friction the critical angle for impending motion can be determined:

Critical angle = $\tan^{-1} u$. (Compare Figure 6 and Table I.)

The Coefficient of Friction

The present model assumes the coefficient of friction to be independent of the normal force and the velocity (compare Shimbo, 1971).

Increasing the coefficient of friction causes a reduction in terminal velocity. On a flat slope u is very important because the normal force is greatest. For increasing angles (and therefore decreasing cos A) the coefficient of friction becomes decreasingly less important. Figure 5 shows the decreasing influence of u on V_t, as a function of the cosine of the angle. Figure 7 gives examples of velocity curves with different u at increasing angles. Figure 8 demonstrates the per-cent increase in terminal velocity caused by a reduction of the coefficient of friction at different angles.

Equations have been derived to compute the influence of u on V_t. As the friction force depends on the normal force and thus on

the steepness of the hill, the angle of the slope has to be considered in the equations:

$$V_1 = V_2 \sqrt{\frac{\sin A - \cos A \, u_1}{\sin A - \cos A \, u_2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$V_2 = \frac{V_1}{\sqrt{\frac{\sin A - \cos A \, u_1}{\sin A - \cos A \, u_2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

The per-cent change in terminal velocity can be calculated with these equations. The result of the computations are summarized in Table II / A-E.

The Mass of the Skier - Ski System

Recalling equation (2);

$$\frac{d}{2} V^2 \, PA \, CD = mg \sin A - mg \cos A \, u$$

we see that an increase in mass will increase the accelerating force and therefore create higher terminal velocities. The effect of a mass increment of V_t has been computed under the assumption that all other parameters (such as planform area) remain constant. The equation cannot be used for inter-individual comparison and prediction of per cent gain in terminal velocity.

The predictive formula reduces to:

$$V_2 = V_1 \sqrt{\frac{\text{mass } 2}{\text{mass } 1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

This indicates that the per-cent gain in V_t depends solely on the base mass and the mass increment. It is irrelevant for the prediction how the terminal velocity was achieved. Table III / A-E summarizes the results of the computations. The tables also show, that

the benefit of adding mass decreases with increasing base mass. Thus a relatively light skier would have the most benefit from carrying extra mass. Figure 9 gives a graphical solution to the same problem. By plotting V_t^2 instead of V_t on the Y axis the hyperbolic function could be changed to a linear relationship. This method is not very efficient as each graph is only true for one specific case.

Aerodynamic Factors

Recalling the equation of drag:

$$\frac{d}{2} V^2 \text{ (PA) (CD)} = \text{DRAG}$$

we see that drag is determined by the velocity of the skier, the planform area of the skier and the coefficient of drag.

The Coefficient of Drag

The coefficient of drag is a measure of how streamlined a body is shaped.

There is no consensus on the absolute value of the CD of a skier. Experimental data, collected by Raine in a windtunnel was utilized, to estimate the CD of a skier in different positions. The experimental data was then compared to calculated drag curves (compare Figure 10). Since the skiers were in identical body positions the intra-individual drag differences were most likely caused by the different racing suits. The comparison of empirical and theoretical drag curves revealed that the variations in total air resistance can be explained by a change in the coefficient of drag from .6 to .8.

The same procedure was used for the skiers in a standing position. In accordance with the literature the CD seems to be unaffected by body position (Thomann, 1976). The CD has also been found independent of the velocity. This suggests a sufficiently high Reynold's number. (Fuetterer, 1960). The computer model thus assumes constant coefficients of drag between .6 and .8.

From the equation of drag it is obvious that the CD becomes more important with increasing velocities. This tendency is illustrated in Figure 11.

The influence of CD on the terminal velocity can be expressed mathematically through the relations:

$$V_1 = V_2 \sqrt{\frac{CD\ 2}{CD\ 1}} \quad \ldots\ldots (8) \quad \text{and}$$

$$V_2 = V_1 \sqrt{\frac{CD\ 1}{CD\ 2}} \quad \ldots\ldots (9)$$

Equation 8 gives the loss in V_t for increasing coefficients of drag. Equation 9 computes the gain in V_t due to a reduction of CD. Table IV can be used to determine the per-cent change in V_t caused by a defined change in CD. This percentage, as demonstrated in equations 8 and 9 is independent from all other variables.

The Planform Area

The planform (cross-sectional) area is subject to substantial inter- and intra-individual variations. Similar to the coefficient of drag the importance of a small planform area (i.e. a good racing position) increases with the velocity. Figure 12 shows the importance of the planform area at different velocities. The influence of PA can be computed with the equations:

$$V_1 = V_2 \sqrt{\frac{PA\ 2}{PA\ 1}} \quad \ldots\ldots (10) \quad \text{and}$$

$$V_2 - V_1 \sqrt{\frac{PA\ 1}{PA\ 2}} \quad \ldots\ldots (11)$$

As these equations are identical to (8) and (9), Table IV can also be used to find the per-cent change in V_t caused by an increase or decrease of the planform area.

Other Influencing Factors

The influence of wind from any direction can be accounted for in the model by substituting the relative velocities of the airflow and the skier into the drag computations.

The head resistance which is the resistance of the snow against the bend-up tip of the ski depends mainly on:

- the density of the snow,
- the specific properties of the snow,
- mass of the skier and gear,
- angle of the slope,
- velocity of the skier,
- area of ski,
- shape and area of the ski tip.

A mathematical representation of these variables would be highly inaccurate and of questionable value for the skier especially considering that all races are done on hardpacked snow.

The density of the air depends on the height above sea level, and is assumed to be 1.25 kg/m^3 as a constant throughout the computations.

Results of the Field Experiment

The results obtained from the analysis of the empirical part of the study support the tendencies suggested by the computer model as far as the positive influence of added mass on the terminal velocity is concerned. The results are summarized in Table A.

TABLE A: Summarized results of the field experiment

All velocities are given in meters per second (m/s)

CONDITION	RUN 1	RUN 2	RANGE	MEAN	DIFFERENCE OF MEANS
Standing	13.18	12.80	.38	12.99	.66
Standing 8 Kg added	13.80	13.49	.31	13.65	
Crouching	14.19	14.29	.10	14.24	.37
Crouching 8 kg added	14.68	14.54	.14	14.61	

TABLES V / A-E contain the raw data obtained from the analysis of the data film using the described setup.

The calculated internal body angles and height of the center of mass indicate that the body position was held consistent from run to run. This implies that the measured differences in velocity were caused by the variable mass of the skier-ski system.

Discussion

The results obtained through the presented model are consistent with the logical relevance of the variables discussed. Fukuoka (1971)

derived a more precise equation of motion for a downhill skier. His formula also considers the lift force and the initial velocity generated through the push from the start gate. However, as far as the terminal velocity is concerned his equation leads to almost identical results as the present model. This can be explained by the fact that the initial velocity does not influence the terminal velocity. Furthermore the lift force is very small because, in a good racing position, the upper body is parallel to the slope and only very small angles of attack occur. Thus it seems that the simplifications made when constructing the model are justified by the comprehensible equations and the 'easy to use' tables. It should however be kept in mind that it takes theoretically an infinite time to reach terminal velocity (Templin, 1968). The real top speed is thus always slightly lower than the calculated terminal velocity.

Increasing the mass of the skier-ski system without changing the planform area will increase the net accelerating force and will thus produce higher terminal velocities. Adding extra mass will only create a net gain if one or more of the following conditions apply:

- the skier's body position is not negatively influenced by the mass increment.
- the skier's stability and his contact to the snow are not disturbed (safety considerations).
- speed is the limiting factor (e.g. on a very flat run).
- a loss caused by the added mass is overcompensated.

Thus adding mass to the skier or ski is a possible way to improve the performance in the downhill event. For the more technical

oriented slalom and giant slalom event less or no net gain is to be expected. The question where extra mass should be attached to the system is still open. Carrying a weight belt around the waist would mean that the skier has to support the extra mass with his legs. Inserting lead soles into the skiboots may reduce the skier's agility and his "feel for the snow". Thus the best solution could be to work with small mass increments of 1 to 2 kg. and to distribute the weight around the center of mass and the lower extremities.

Due to the stated limitations, the possibilities for interpretation or discussion of the results of the field experiment are limited. However, the data indicates that an increase in velocity can be achieved by adding mass to the system.

Chapter V

Summary and Recommendations

The purpose of this study was to quantify the influence of selected variables on terminal velocity in downhill skiing.

A model of selected variables (angle of the slope, coefficient of friction, mass of the skier-ski system, Planform area and coefficient of drag) influencing the terminal velocity of a skier skiing down a uniform slope in a uniform position has been presented. The basic equation derived allows prediction of terminal velocity for specified cases. The importance of the single factors for the dependent variable (terminal velocity) has been quantified. In most cases input variables and results have been compared and validated using the related literature. The results of a field experiment investigating the influence of added mass on the terminal velocity for different skiing positions support the tendencies predicted from the mechanical model.

The angle of the slope is the most important single factor determining terminal velocity. The critical angle where motion is impending and the presented equation of motion becomes true depends on the coefficient of friction between ski and snow.

The coefficient of friction is very important at low angles and loses influence on steeper hills as a function of the cosine of the angle. Equations describing the importance of the coefficient of frictionhave been derived.

The mass of the skier-ski system has a positive influence on the terminal velocity; the skier with the higher body density or

added mass has an advantage. Formulas describing the gain due to a defined mass increment have been derived.

As drag increases exponentially with velocity, factors determining total air resistance become more important as the velocity increases. Mathematical expressions to quantify the importance of the planform area and the coefficient of drag have been derived.

The equations derived were used to produce tables that allow the prediction of the change in terminal velocity caused by a defined change in one of the variables. A graphical model illustrating the relations and inter-relations of the variables discussed is presented. A field study should be devised to determine whether the conditions for a net gain in time, stated in the previous section, are fulfilled under racing conditions.

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APPENDIX

APPENDIX A

FIGURES

FIGURE 1: The Velocity of the Skier disregarding Air Resistance.

Dashed lines: Velocity at time t assuming a coefficient of friction of .05.

Solid lines: Velocity at time t assuming a coefficient of firction of .00.

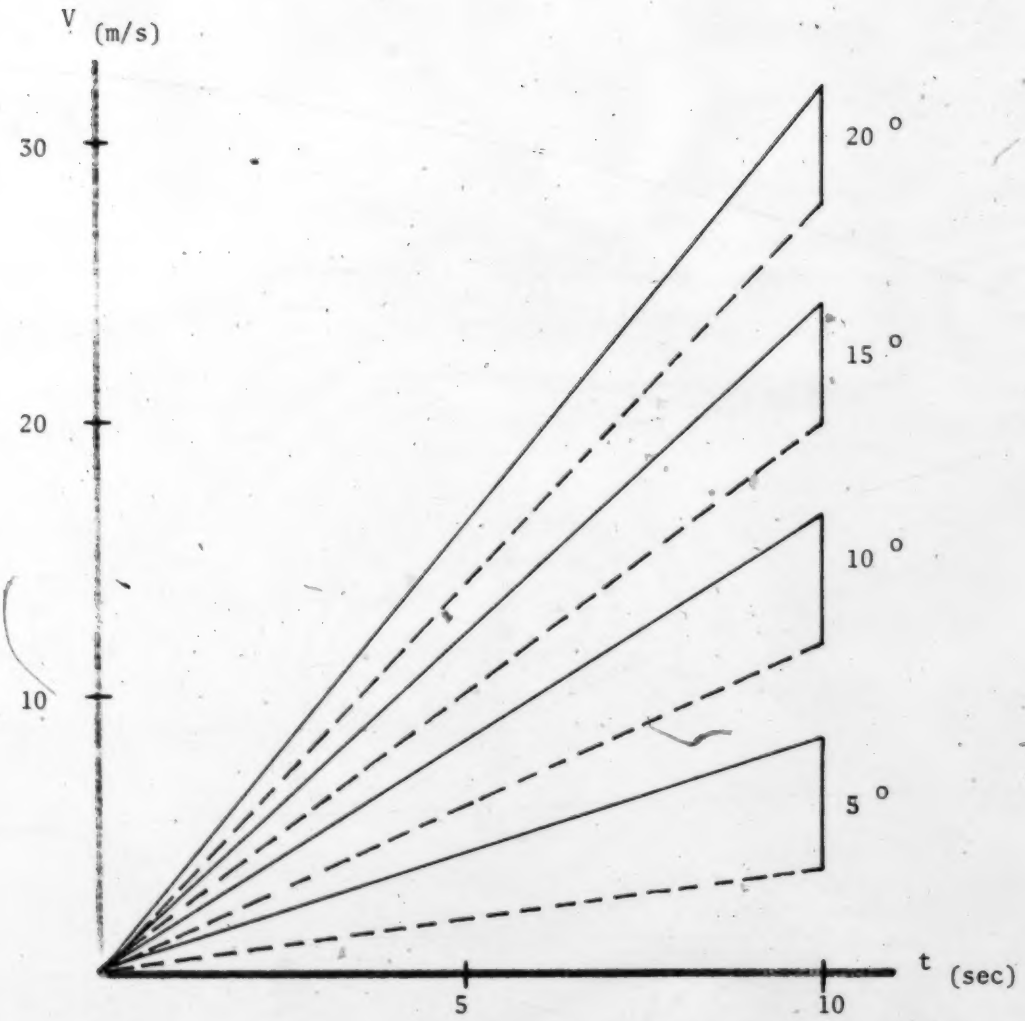


FIGURE 2: Velocity Curves for varying Coefficients of Friction.

Input: Mass = 80 kg; Angle of slope = 15 ^{o};
Planform area = .6 m^2; Coefficient of drag = .7.

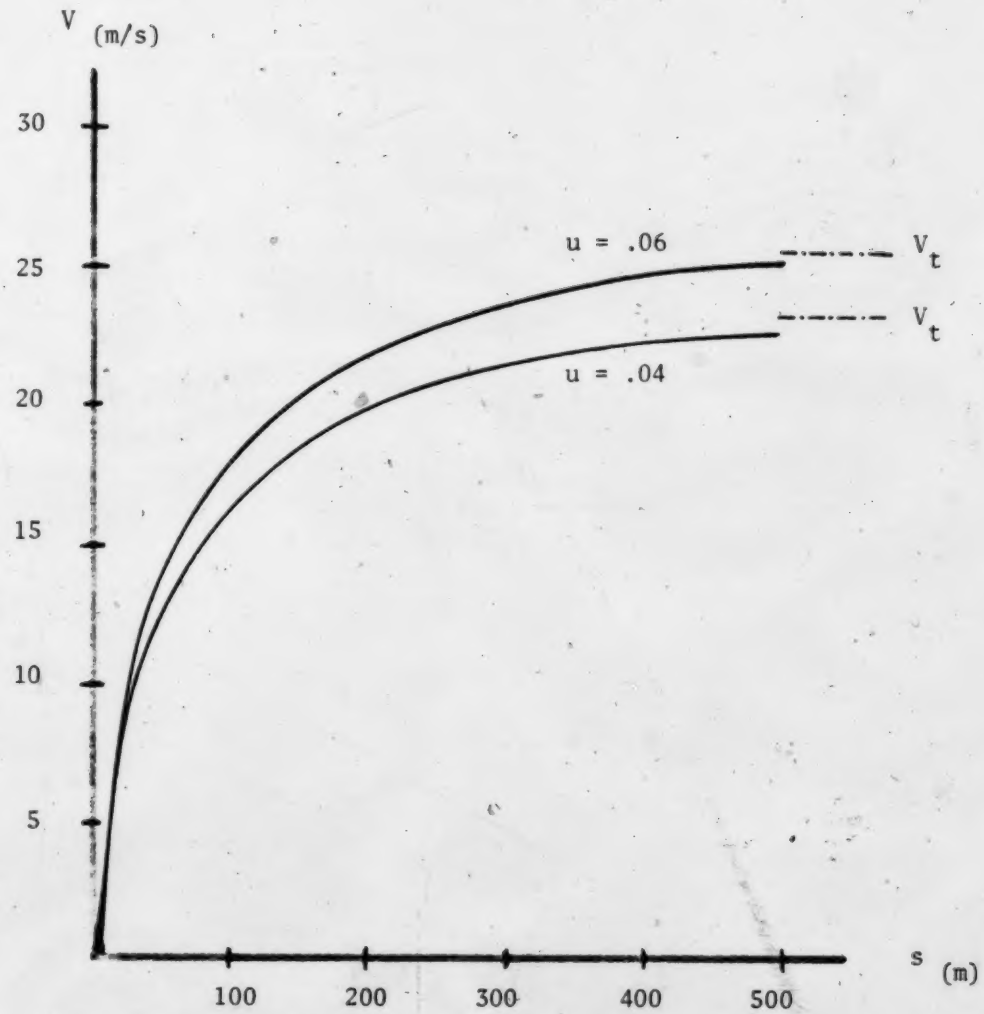


FIGURE 3: Velocity Curves for varying Coefficients of Drag.

Input: Mass = 80 kg; Angle of slope = 15 ^{o};
Planform area = .5 m^2; Coefficient of friction = .05.

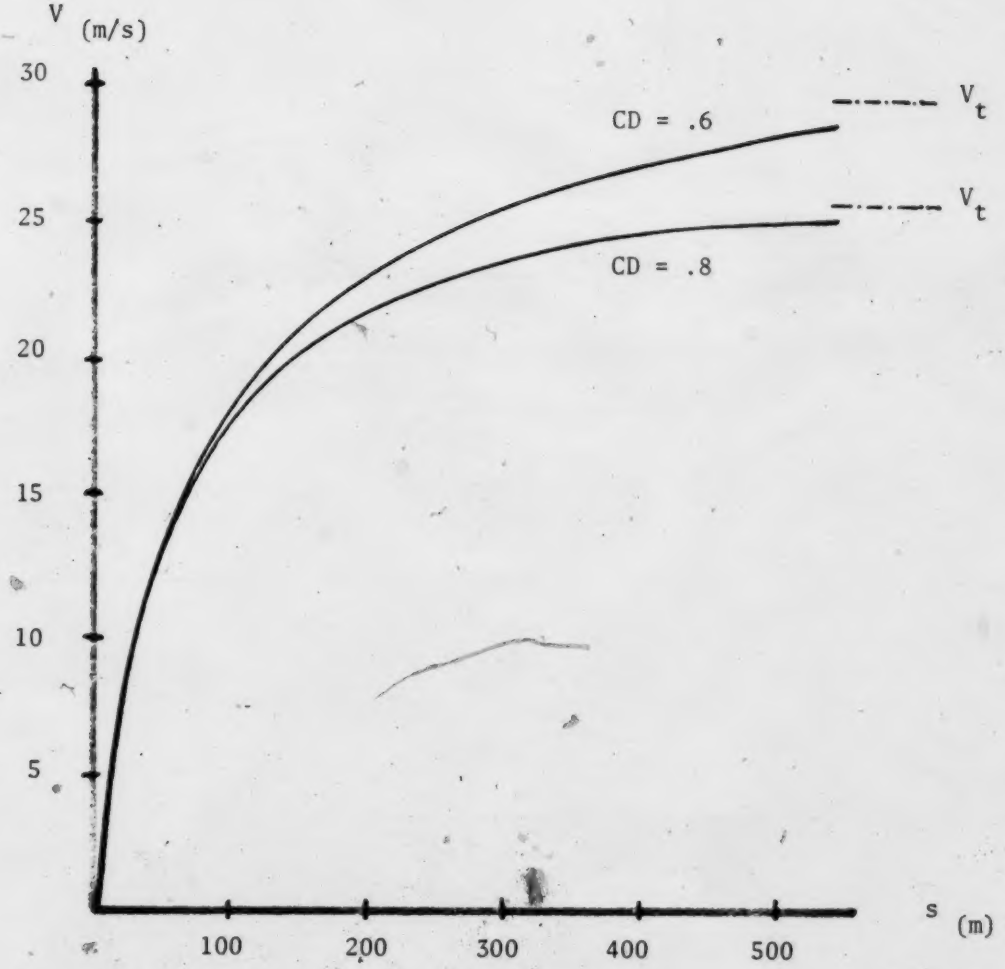


FIGURE 4: Velocity Curves for extreme Input Values.

Input: Mass = 80 kg; Angle of slope = 15 °

Curve A: u = .01; PA = 1.2 m^2; CD = .9.

Curve B: u = .20; PA = .3 m^2; CD = .6.

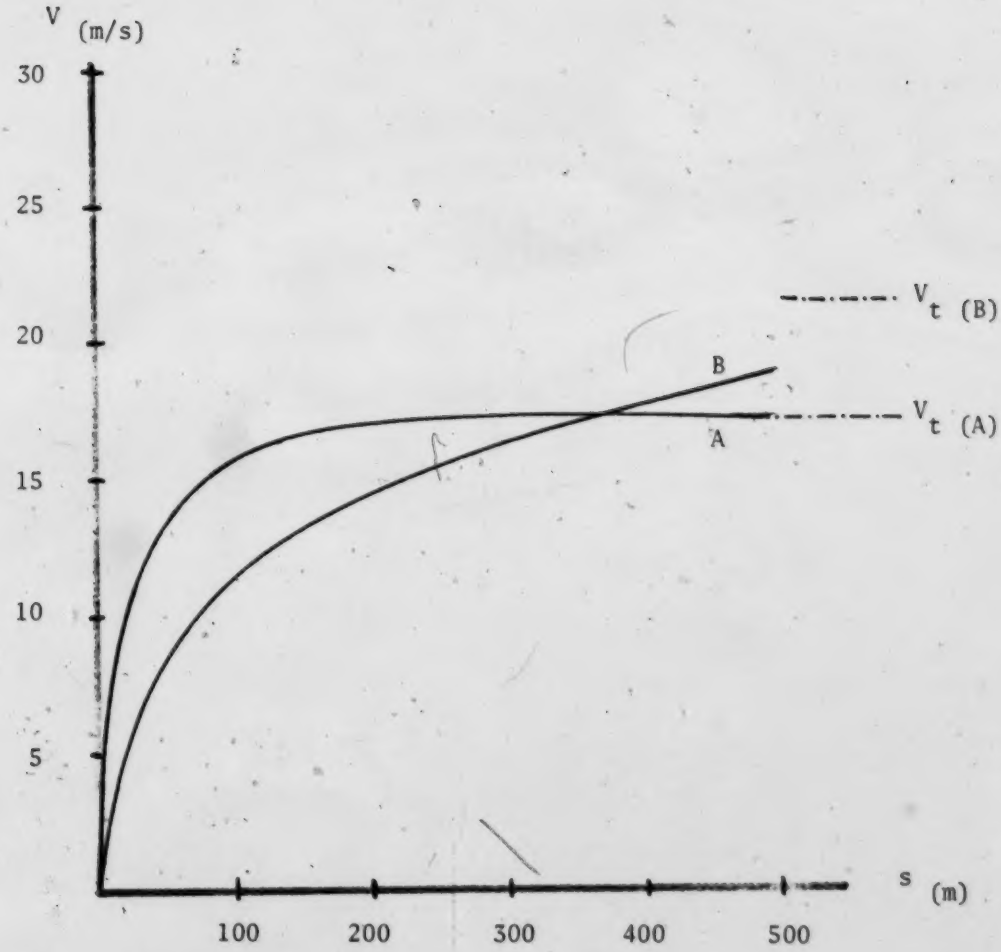


FIGURE 5: Decreasing influence of the Coefficient of Friction as a Function of the Cosine of the Angle.

FIGURE 6: Graphical Determination of the critical Value for the Angle of the Slope.

For a given coefficient of friction (u) motion is impending at the point of intersection with $f(x) = \sin x$

FIGURE 7: Influence of the Coefficient of Friction on the Terminal Velocity at different Angles of the Slope.

Input: Mass = 80 kg; Planform Area = .6 m^2; Coefficient of Drag = .8.

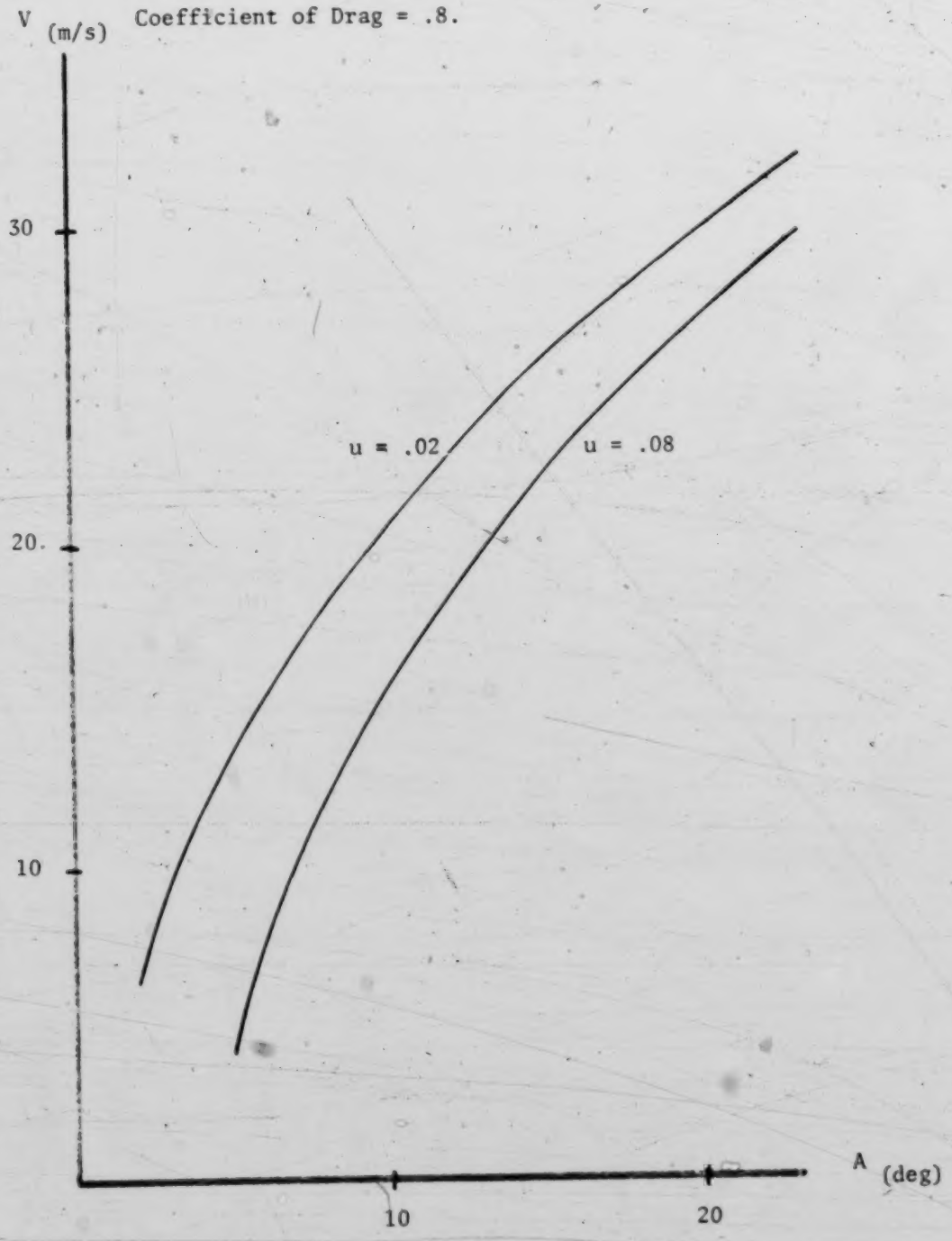


FIGURE 8: Influence of a decreased Coefficient of Friction on the Terminal Velocity at different Angles of the Slope.

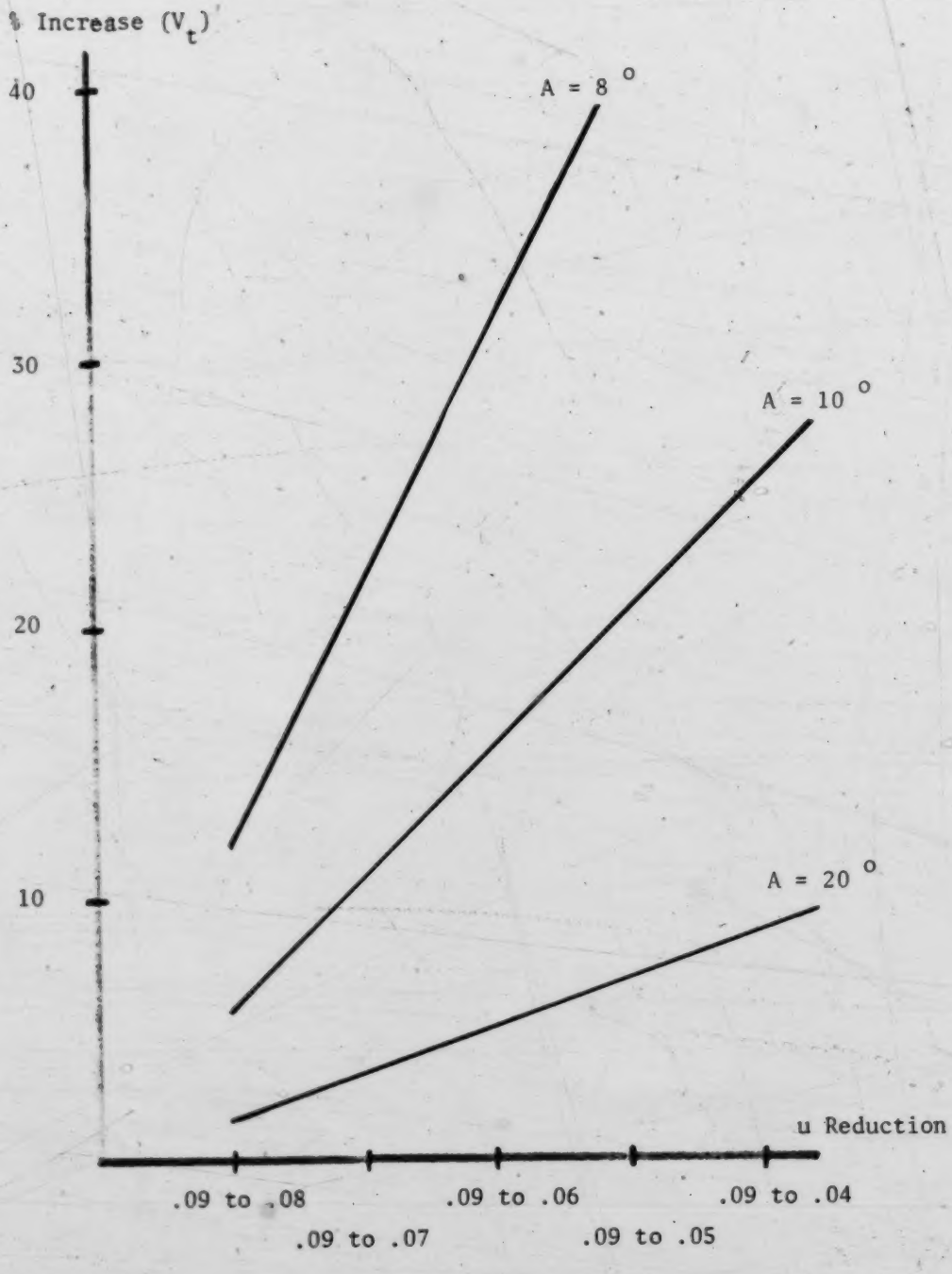


FIGURE 9: The Influence of Mass on Terminal Velocity.

The gain in terminal velocity due to an arbitrary mass increment can be read from the right vertical axis.
Input: Planform area = .6 m^2; Coefficient of drag = .8; Coefficient of friction = .05.

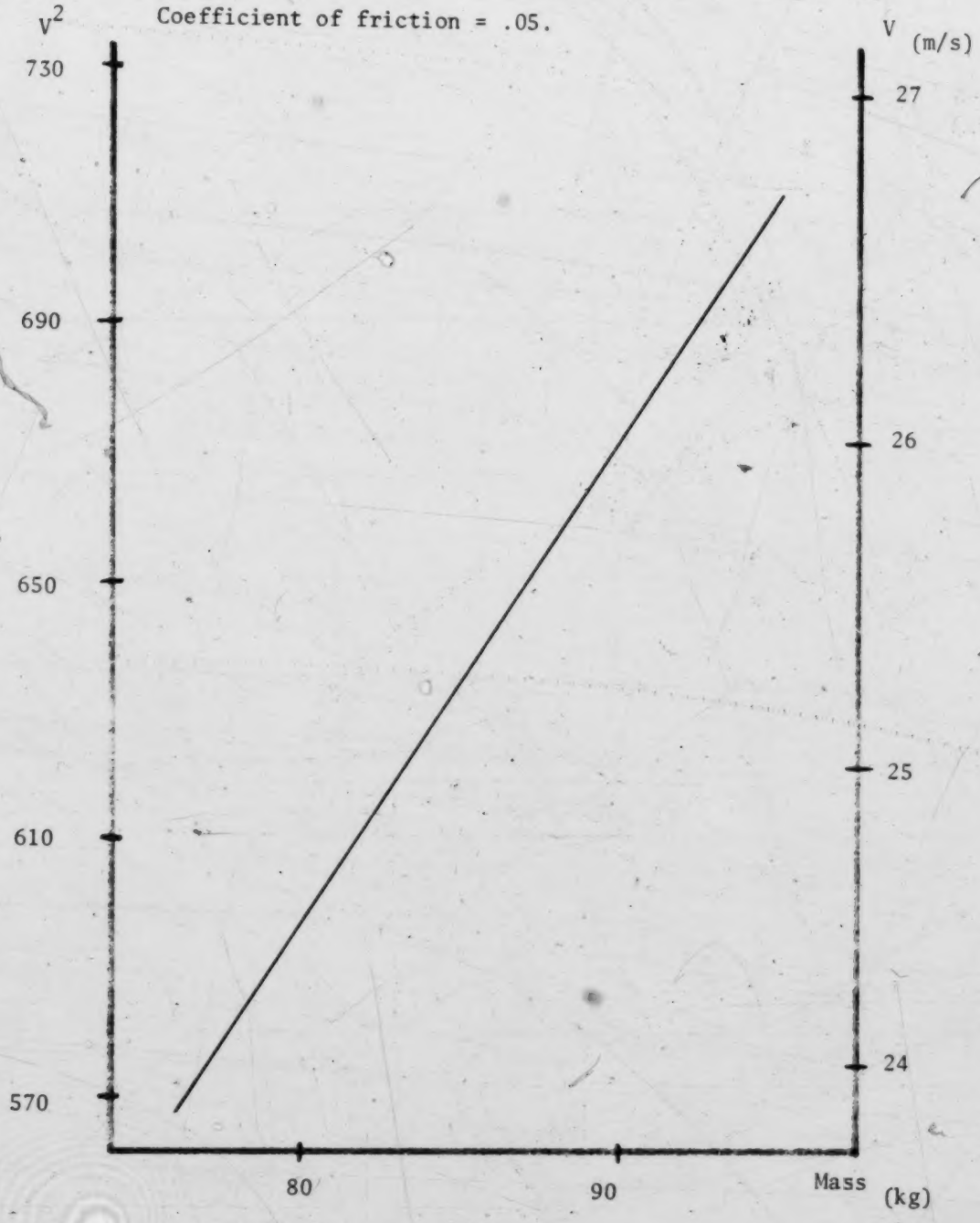


FIGURE 10: Validation of the assumed Coefficient of Drag Values.

Dashed lines: Calculated drag curves.
Solid lines: Empiric data (Windtunnel).

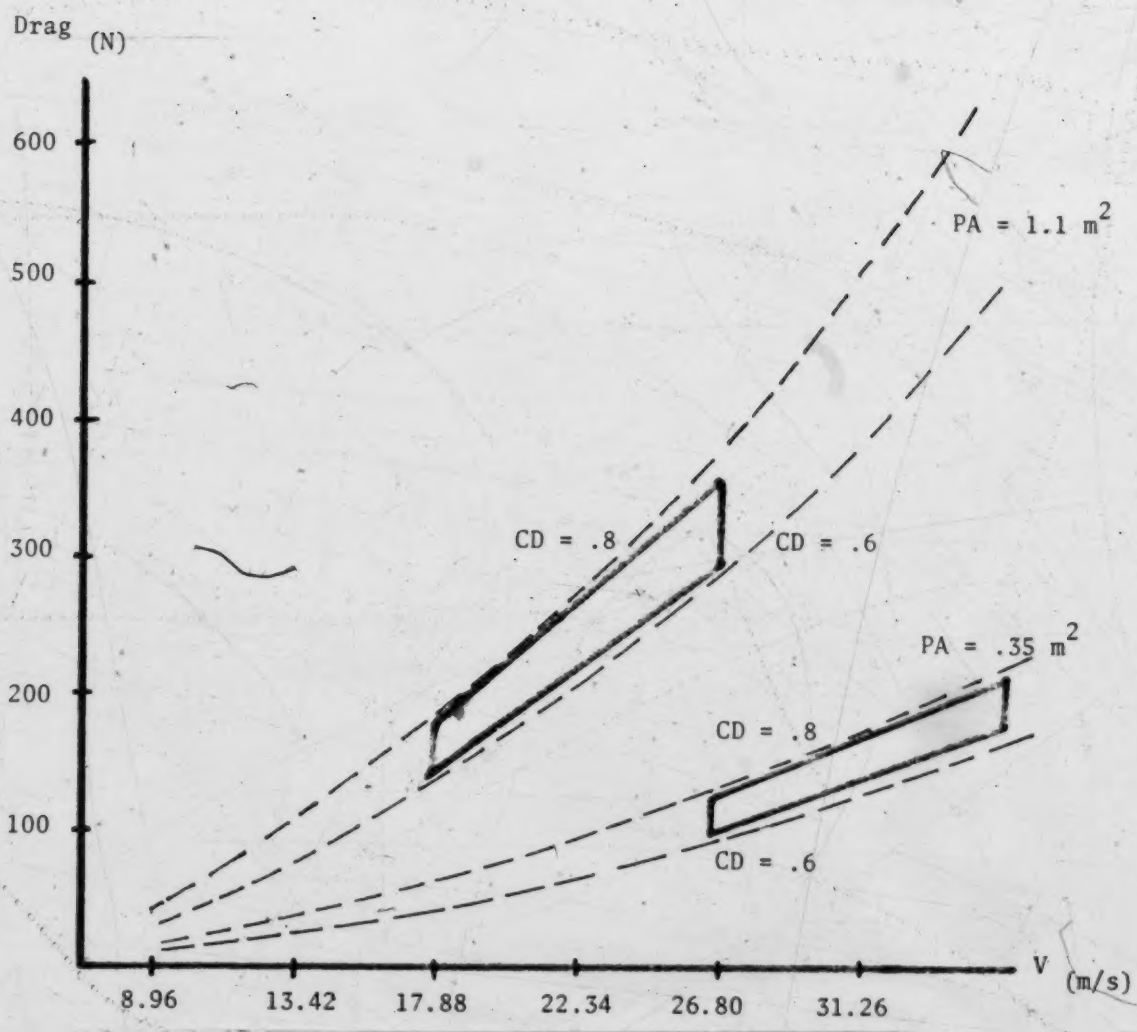


FIGURE 11: Influence of the coefficient of Drag on the Terminal Velocity.

Input: Mass = 80 kg; Planform area = .6 m^2
Coefficient of friction = .05.

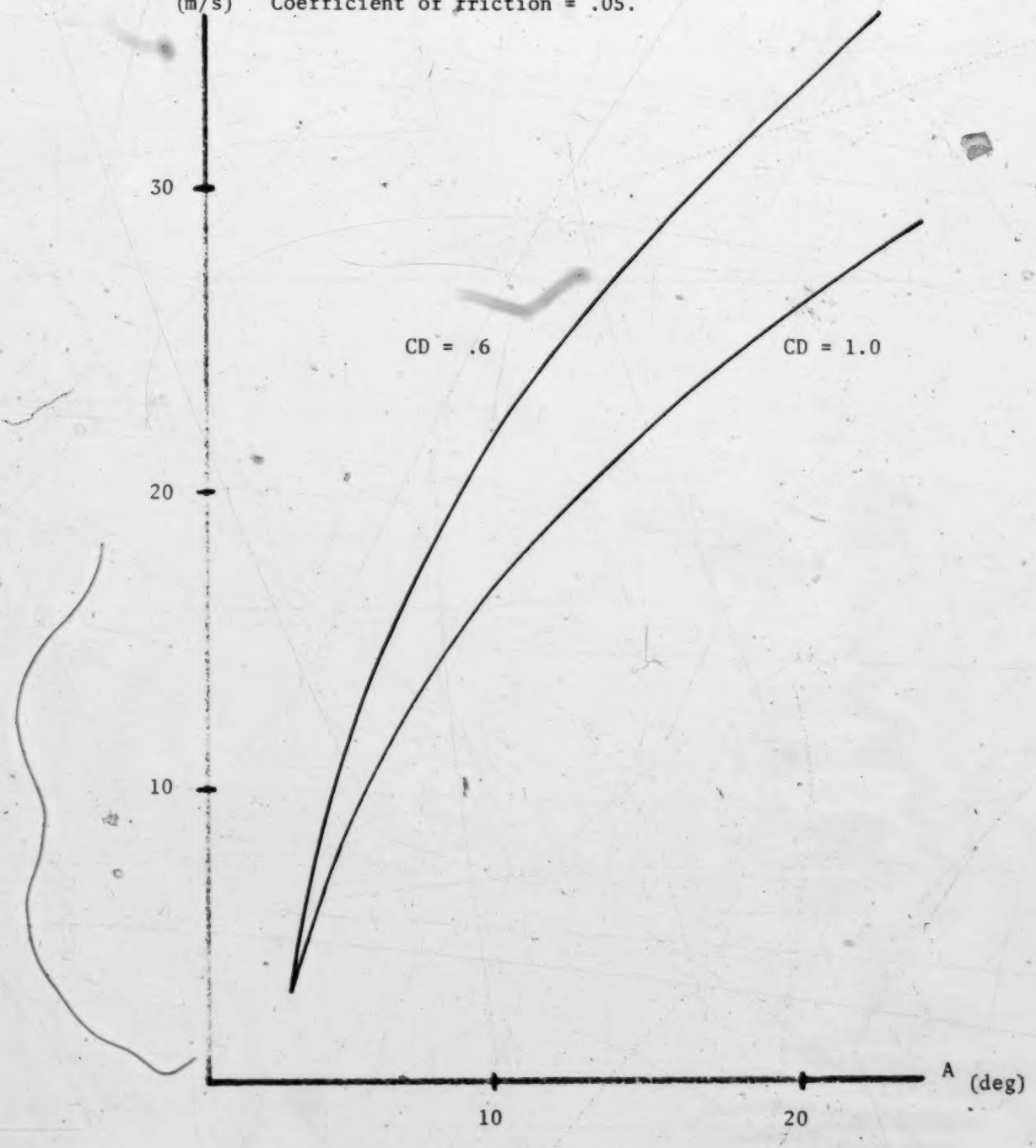


FIGURE 12: Influence of the Planform Area on the Terminal Velocity.

Input: Mass = 80 kg; Coefficient of drag = .8;
Coefficient of friction = .05.

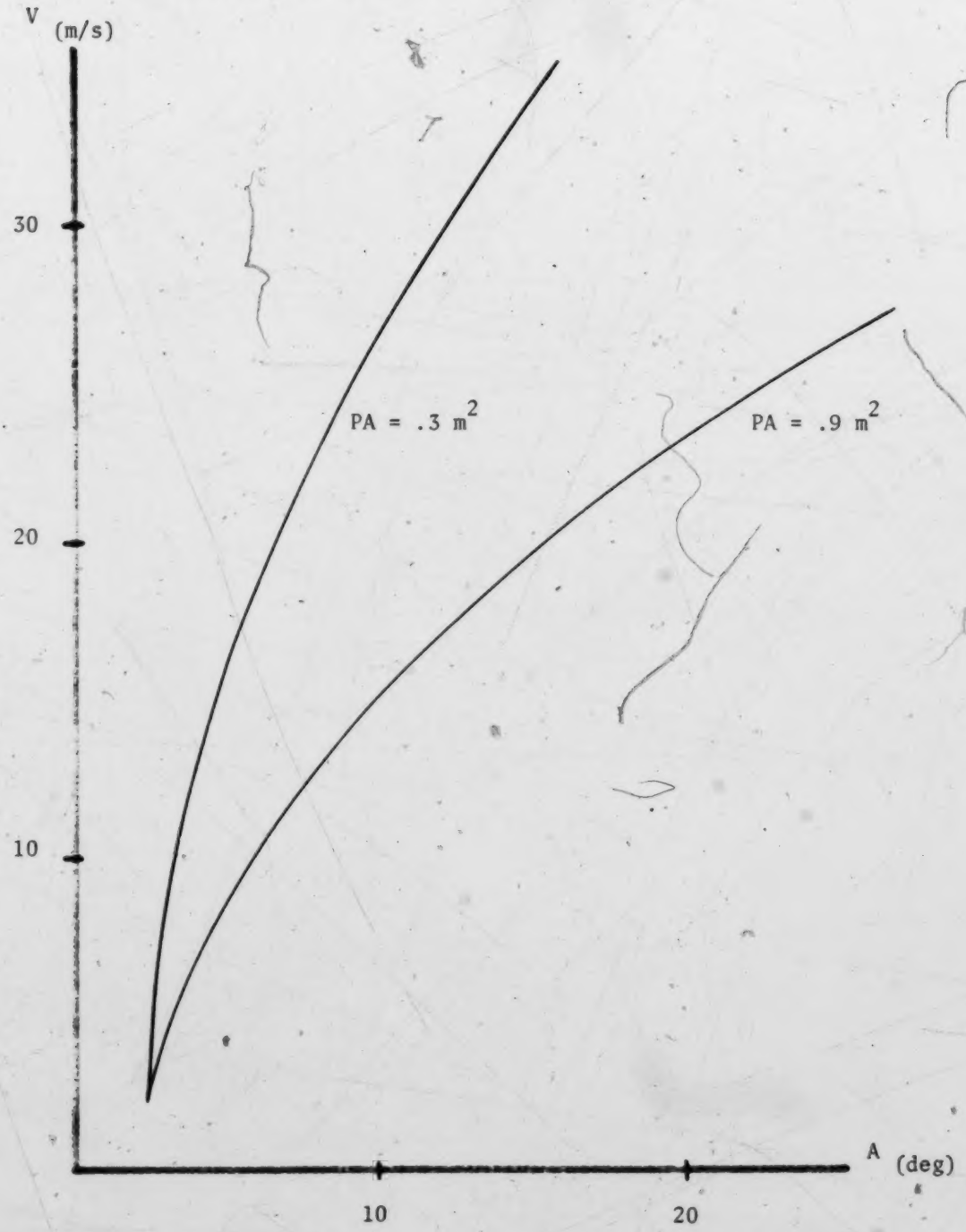
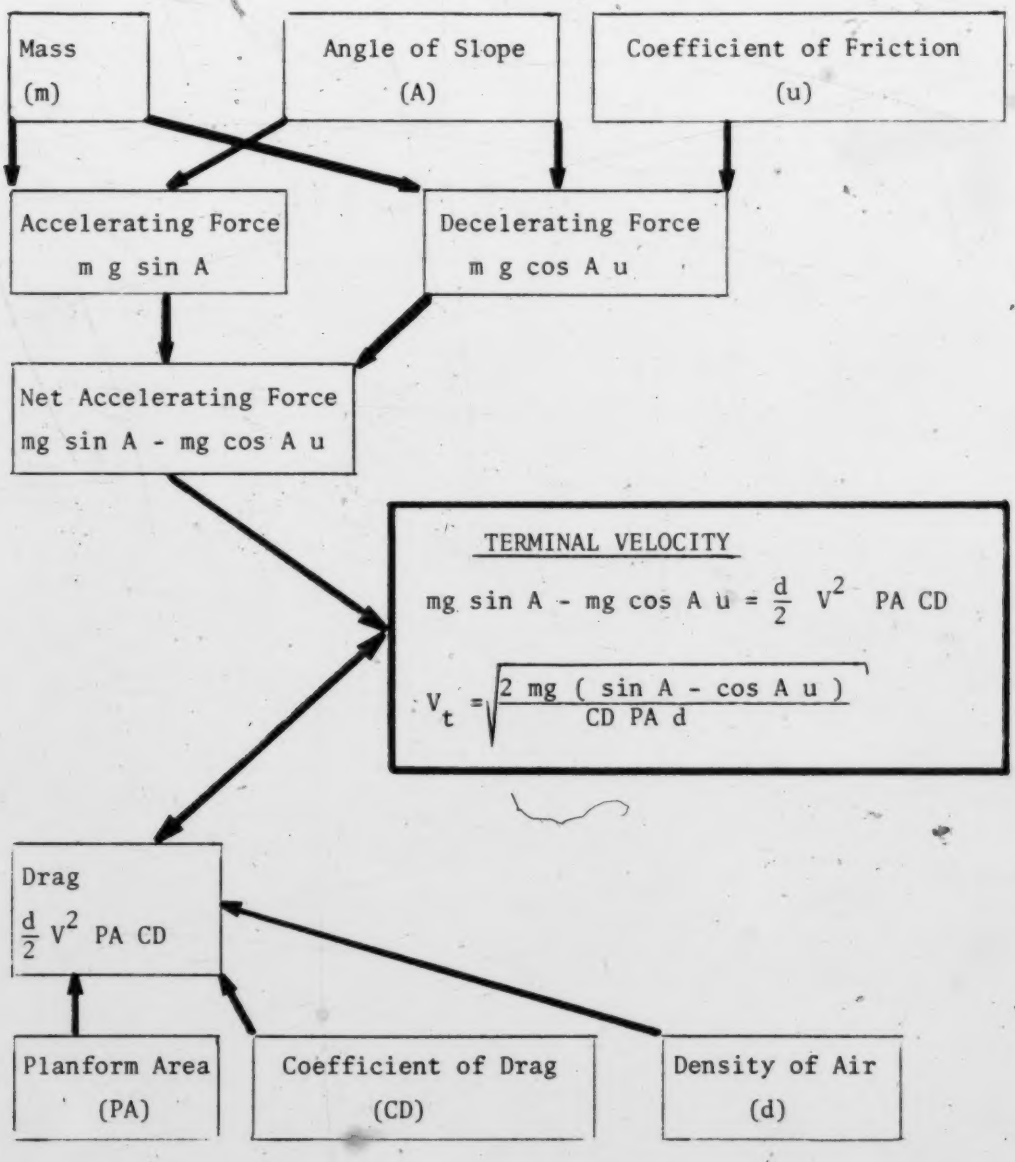


FIGURE 13: Graphical Model of the Relations and Interrelations of selected Variables influencing Terminal Velocity.



APPENDIX B

TABLES

TABLE I:

Critical values of a for impending motion for a given coefficient of friction

u	A_c (deg)
.01	.5729
.02	1.1458
.03	1.7184
.04	2.2906
.05	2.8624
.06	3.4334
.07	4.0042
.08	4.5739
.09	5.1428
.10	5.7106

Example: If the coefficient of friction between ski and snow is .06 the skier needs a slope of 3.4334° to start sliding.

EXPLANATIONS TO TABLE II/A-E

Table to Calculate the Influence of the Coefficient of Friction on Terminal Velocity

Use: To find the change in terminal velocity caused by an increase or decrease of the coefficient of friction (u).

Procedure (1): Decrease of the C of friction:

1. Find the table for the desired angle of the slope.
2. Find the original u in the leftmost column.
3. Find the REDUCED u in the upper row.
4. Read off the per-cent increase in V_t.
5. Original V_t + % increase of VT = New terminal velocity.

   Example: A skier on a 25° slope achieves a terminal velocity of 32.75 m/s. By reducing u from .05 to .02 he will increase his V_t by 3.54% to 33.91 m/s.

Procedure (2): Increase of the C of friction:

1. Find the original u in the leftmost column.
2. Find the INCREASED C of friction in the top row.
3. The new (reduced) velocity can be obtained by multiplying the original V_t by the factor indicated. (Factor x 100 = % increase from original V_t).

   Example: A skier on a 30° slope will reduce his V_t from 11 m/s. to 10.39 m/s. when increasing u from .05 to .07.

TABLE II /A: The influence of the coefficient of friction on the terminal velocity

ANGLE: 30 degrees

	.10	.09	.08	.07	.06	.05	.04	.03	.02	.01
.10	-	1.042	2.073	3.094	4.106	5.107	6.099	7.081	8.055	9.020
.09	.990	-	1.021	2.031	3.032	4.023	5.005	5.977	6.941	7.896
.08	.980	.990	-	1.000	1.991	2.972	3.944	4.906	5.860	6.806
.07	.970	.980	.990	-	.981	1.952	2.914	3.867	4.812	5.748
.06	.961	.971	.980	.990	-	.962	1.915	2.859	3.794	4.721
.05	.951	.961	.971	.981	.990	-	.944	1.879	2.805	3.723
.04	.943	.952	.962	.972	.981	.991	-	.926	1.844	2.754
.03	.934	.944	.953	.963	.972	.982	.991	-	.909	1.811
.02	.925	.935	.945	.954	.963	.973	.982	.991	-	.893
.01	.917	.927	.936	.946	.955	.964	.973	.882	.991	-

TABLE II /B: The influence of the coefficient of friction on the terminal velocity

ANGLE: 25 degrees

	.10	.09	.08	.07	.06	.05	.04	.03	.02	.01
.10	-	1.356	2.694	4.014	5.318	6.607	7.879	9.137	10.381	11.611
.09	.987	-	1.320	2.623	3.910	5.181	6.436	7.677	8.904	10.118
.08	.974	.987	-	1.236	2.556	3.810	5.050	6.275	7.486	8.683
.07	.961	.974	.987	-	1.254	2.492	3.716	4.925	6.121	7.303
.06	.950	.962	.975	.988	-	1.223	2.432	3.626	4.807	5.974
.05	.938	.951	.963	.976	.988	-	1.194	2.374	3.540	4.694
.04	.927	.940	.952	.964	.976	.988	-	1.166	2.319	3.459
.03	.916	.929	.941	.953	.965	.977	.988	-	1.139	2.266
.02	.906	.918	.930	.942	.954	.966	.977	.989	-	1.114
.01	.896	.908	.920	.932	.944	.955	.967	.978	.989	-

TABLE II /C: The influence of the coefficient of friction on the terminal velocity

ANGLE: 20 degrees

	.10	.90	.08	.07	.06	.05	.04	.03	.02	.01
.10	-	1.877	3.719	5.530	7.309	9.060	10.783	12.480	14.152	15.799
.09	.982	-	1.809	3.586	5.333	7.051	8.743	10.408	12.049	13.666
.08	.964	.982	-	1.746	3.462	5.150	6.811	8.447	10.059	11.647
.07	.948	.965	.983	-	1.687	3.346	4.979	6.586	8.170	9.732
.06	.932	.949	.967	.983	-	1.632	3.237	4.819	6.376	7.912
.05	.917	.934	.951	.968	.984	-	1.580	3.136	4.669	6.179
.04	.903	.920	.936	.953	.969	.984	-	1.532	3.040	4.528
.03	.889	.906	.922	.938	.954	.970	.985	-	1.486	2.951
.02	.876	.892	.909	.924	.940	.955	.970	.985	-	1.443
.01	.864	.880	.896	.911	.927	.942	.957	.971	.986	-

TABLE II /D: The influence of the coefficient of friction on the terminal velocity

ANGLE: 15 degrees

	.10	.09	.08	.07	.06	.05	.04	.03	.02	.01
.10	-	2.934	5.787	8.565	11.273	13.917	16.501	19.029	21.505	23.930
.09	.971	-	2.771	5.470	8.101	10.670	13.180	15.636	18.041	20.398
.08	.945	.973	-	2.626	5.186	7.686	10.128	12.518	14.858	17.151
.07	.921	.948	.974	-	2.495	4.930	7.310	9.639	11.919	14.154
.06	.899	.925	.951	.976	-	2.376	4.698	6.970	9.195	11.375
.05	.878	.904	.929	.953	.977	-	2.268	4.488	6.661	8.790
.04	.858	.884	.908	.932	.955	.978	-	2.170	4.295	6.377
.03	.840	.865	.889	.912	.935	.957	.979	-	2.080	4.118
.02	.828	.847	.871	.894	.916	.938	.959	.980	-	1.997
.01	.807	.831	.854	.876	.898	.919	.940	.960	.980	-

TABLE II /E: The influence of the coefficient of friction on the terminal velocity

ANGLE: 10 degreés

	.10	.09	.08	.07	.06	.05	.04	.03	.02	.01
.10	-	6.349	12.340	18.027	23.453	28.650	33.645	38.460	43.113	47.619
.09	.940	-	5.633	10.981	16.083	20.969	25.666	30.193	34.569	38.806
.08	.890	.947	-	5.063	9.892	14.518	18.964	23.250	27.392	31.404
.07	.847	.901	.952	-	4.597	9.000	13.232	17.311	21.254	25.072
.06	.810	.861	.910	.956	-	4.210	8.256	12.156	15.925	19.575
.05	.777	.827	.873	.917	.960	-	3.883	7.625	11.242	14.745
.04	.748	.796	.841	.883	.924	.963	-	3.361	6.615	6.615
.02	.699	.743	.785	.825	.863	.899	.934	.967	-	3.149
.01	.677	.720	.761	.800	.836	.871	.905	.938	.969	-

EXPLANATIONS TO TABLE III/A-E

Table to Determine % Increase in Terminal Velocity Due to A Defined Mass Increment

Procedure:

1. Find the mass of the skier - ski system in the leftmost column (Kg).
2. Find the new mass of the system (mass + mass increment) in the uppermost row.
3. Read the % increase in terminal velocity off the table.
4. Obtain the new terminal velocity by adding the indicated per-cent gain to the previously attained terminal velocity.
5. The resulting gain in time can be calculated as

$$\frac{\text{length of run}}{\text{old velocity}} - \frac{\text{length of run}}{\text{new velocity}} = \text{TIME GAIN (estimate)}$$

Note:

The skier does not ski the entire course at terminal velocity. Thus the actual time gain will be slightly lower than the calculated value.

Example:

A 74 kg. skier (with equipment) attains (on a certain slope) a terminal velocity of 30 m/s. By adding one kg. mass he will increase his velocity by .669% to 30.2 m/s.*

---

* To convert from m/s. (meters per second) to km/h multiply by 3.6.

His time gain on a 2500 m. course can be obtained as:

$$\frac{2500}{30.0} - \frac{2500}{30.2} = .55 \text{ seconds}$$

TABLE III /A: Table to determine % increase in terminal velocity due to a defined mass increment from a given basemass

	61	62	63	64	65	66	67	68	69	70	71	72	73	74
60	.823	1.626	2.410	3.175	3.923									
61		.810	1.600	2.372	3.126	3.862								
62			.797	1.575	2.335	3.078	3.804							
63				.784	1.550	2.299	3.031	3.747						
64					.772	1.527	2.264	2.986	3.691					
65						.760	1.504	2.231	2.942	3.638				
66							.749	1.482	2.198	2.899	3.585			
67								.738	1.460	2.166	2.858	3.485		
68									.727	1.439	2.135	2.817	3.485	
69										.717	1.419	2.105	2.778	3.437

TABLE III /B: Table to determine % increase in terminal velocity due to a defined mass increment from a given basemass

	71	72	73	74	75	76	77	78	79	80	81	82	83	84
70	.707	1.399	2.076	2.740	3.391									
71		.697	1.379	2.0448	2.703	3.345								
72			.687	1.361	2.020	2.667	3.301							
73				.678	1.342	1.994	2.632	3.258						
74					.669	1.325	1.967	2.598	3.216					
75						.660	1.307	1.942	2.565	3.176				
76							.651	1.290	1.917	2.532	3.136			
77								.643	1.274	1.893	2.500	3.097		
78									.635	1.258	1.869	2.470	3.059	
79										.627	1.242	1.846	2.439	3.022

TABLE III /C: Table to determine % increase in terminal velocity due to a defined mass increment from a given basemass

	81	82	83	84	85	86	87	88	89	90	91	92	93	94
80	.619	1.227	1.824	2.410	2.986									
81		.612	1.212	1.802	2.381	2.951								
82			.604	1.198	1.781	1.353	2.916							
83				.597	1.183	1.76	2.326	2.882						
84					.590	1.170	1.739	2.299	2.850					
85						.583	1.156	1.719	2.273	2.817				
86							.576	1.143	1.700	2.247	2.786			
87								.570	1.130	1.681	2.222	2.755		
88									.563	1.117	1.662	2.198	2.725	
89										.557	1.105	1.644	2.174	2.696

TABLE III /D: Table to determine % increase in terminal velocity due to a defined mass increment from a given basemass

	91	92	93	94	95	96	97	98	99	100	101	102	103	104
90	.551	1.093	1.626	2.151	2.667									
91		.545	1.081	1.609	2.128	2.639								
92			.539	1.070	1.592	2.105	2.611							
93				.533	1.058	1.575	2.084	2.584						
94					.528	1.047	1.559	2.062	2.558					
95						.522	1.036	1.543	2.041	2.532				
96							.517	1.026	1.527	2.020	2.507			
97								.512	1.015	1.511	2.000	2.482		
98									.506	1.005	1.496	1.980	2.457	
99										.501	.995	1.482	1.961	2.433

TABLE III /E: Table to determine % increase in terminal velocity due to a defined mass increment from a given basemass

	101	102	103	104	105	106	107	108	109	110	111	112	113	114
100	.496	.985	1.467	1.942	2.410									
101		.491	.976	1.453	1.923	2.387								
102			.487	.966	1.439	1.905	2.364							
103				.482	.957	1.425	1.887	2.342						
104					.477	.948	1.412	1.869	2.321					
105						.473	.939	1.399	1.852	2.299				
106							.468	.930	1.386	1.835	2.278			
107								.464	.922	1.373	1.818	2.258		
108									.460	.913	1.361	1.802	2.237	
109										.456	.905	1.348	1.786	2.218

EXPLANATIONS TO TABLE IV

Table to Calculate the Influence of the Planform Area and the Coefficient of Drag on the Terminal Velocity

Use: To find the change in terminal velocity caused by an increase or decrease in Planform area or C of Drag.

Procedure (1): Decrease of PA or CD:

1. Find the skier's PA or CD in the leftmost column.
2. Find the REDUCED PA or CD in the top row.
3. Read off the per-cent increase from the original V_t.

   Example: A skier achieving a V_t of 15 m/s. with a PA of .9 m^2 will increase his velocity by 13.39% to 17.01 m/s. when reducing the PA to .7 m^2.

Procedure (2): Increase of PA or CD:

1. Find the skier's original PA or CD in the leftmost column.
2. Find the INCREASED PA or CD in the top row.
3. The reduced V_t is given as the per-cent of the original V_t.

   Example: A skier achieving a V_t of 22 m/s. increases his coefficient of drag from .6 to .9. His new velocity = 81.65% of the original V_t = 17.96 m/s.

TABLE IV: Table to calculate the influence of the planform area and the coefficient of drag on the terminal velocity

	1.2	1.1	1.0	.9	.8	.7	.6	.5	.4	.3
1.2	-	4.45	9.54	15.47	22.47	30.93	41.42	54.92	73.21	100
1.1	95.47	-	4.88	10.55	17.26	25.36	35.40	48.32	65.83	91.49
1.0	91.29	95.35	-	5.41	11.80	19.52	29.10	41.42	58.11	82.57
.9	86.60	90.45	94.87	-	6.07	13.39	22.47	34.16	50.00	73.21
.8	81.65	85.28	89.44	94.28	-	6.90	15.47	26.49	41.42	63.30
.7	76.38	79.77	83.67	88.19	93.54	-	8.01	18.32	32.29	52.75
.6	70.71	73.85	77.46	81.65	86.60	92.58	-	9.54	22.47	41.42
.5	64.55	67.42	70.71	74.54	79.06	84.52	91.29	-	11.80	29.10
.4	57.74	60.30	63.25	66.67	70.71	75.59	81.65	89.44	-	15.47
.3	50.00	52.22	54.77	57.74	61.24	65.47	70.71	77.46	86.60	-

TABLE V /A: Results of the analysis of the field experiment

The skier in a relaxed standing position.

```
++++++++++++++++

RUN #            1

TIME/FRAME
.0214
# FRAMES
5

----------------
X AND Y COORDINA
TES OF C/M
          24.3686
           6.3119

HEIGHT C/M:
          77.9455
----------------

----------------
X AND Y COORDINA
TES OF C/M
          36.2847
           6.4095

HEIGHT C/M:
          79.153
----------------

AVERAGE VELOCIT
 OF C/M BETWEEN
FRAME

Vh (m/s)=  13.1
Vv (m/s)=   0.1
Vlin(m/s)= 13.1

Angle resultant
           0.4
```

```
RUN #            2

Time/Frame
.0213

++++++++++++++++

++++++++++++++++

INTERNAL ANGLES
     (Deg.)

++++++++++++++++

FRAME #          1

HIP            143
KNEE           140
L.LEG           74

------------

FRAME #          2

HIP            147
KNEE           139
L.LEG           76

------------

++++++++++++++++

   VELOCITIES

++++++++++++++++

F intervol
5

Vel. F#          1
                 2
m/s          12.80
```

TABLE V./B: Results of the analysis of the field experiment

The skier in a relaxed standing position.

MASS INCREMENT : 8 kg

```
++++++++++++++++

RUN #            4

TIME/FRAME
.0213
# FRAMES
5

-----------------
X AND Y COORDINA
TES OF C/M
          25.0856
           6.0637

HEIGHT C/M:
          74.8868
-----------------

-----------------
X AND Y COORDINA
TES OF C/M
          36.9882
           6.2238

HEIGHT C/M:
          76.8639
-----------------

AVERAGE VELOCITY
 OF C/M BETWEEN
FRAME     1.0000
          2.0000
Vh (m/s)=  13.80
Vv (m/s)=   0.19
Vlin(m/s)= 13.80

Angle resultant
           0.77
```

```
RUN #.           7

Time/Frame
.0213

++++++++++++++++

++++++++++++++++

INTERNAL ANGLES
    (Deg.)

++++++++++++++++

FRAME #          1

HIP            140
KNEE           134
L.LEG           73

  ------------

FRAME #          2

HIP            143
KNEE           134
L.LEG           75

  ------------

++++++++++++++++

   VELOCITIES

++++++++++++++++

F interval
5

Vel. F#          1
                 2
m/s            13.49
```

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TABLE V /C: Results of the analysis of the field experiment

The skier in a crouched position (racing position)

RUN # 2	RUN # 4
TIME/FRAME .0212	TIME/FRAME .0212
# FRAMES 5	# FRAMES 5
X AND Y COORDINATES OF C/M 28.4279 3.9472	X AND Y COORDINATES OF C/M 17.9416 4.5373
HEIGHT C/M: 48.7484	HEIGHT C/M: 56.0357
X AND Y COORDINATES OF C/M 30.6064 3.8175	X AND Y COORDINATES OF C/M 30.2091 4.4918
HEIGHT C/M: 47.1464	HEIGHT C/M: 55.4741
AVERAGE VELOCITY OF C/M BETWEEN FRAME 1 2	AVERAGE VELOCITY OF C/M BETWEEN FRAME 1.0000 2.0000
Vh (m/s)= 14.19	Vh (m/s)= 14.29
Vv (m/s)= -0.15	Vv (m/s)= -0.05
Vlin(m/s)= 14.19	Vlin(m/s)= 14.29
Angle resultant -0.61	Angle resultant -0.21

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TABLE V /D: Results of the analysis of the field experiment

The skier in a crouched position (racing position)

MASS INCREMENT : 8 kg

```
++++++++++++++++

RUN #          8

TIME/FRAME
.0214
# FRAMES
5

----------------
X AND Y COORDINA
TES OF C/M
          25.7369
           3.6543

HEIGHT C/M:
          45.1307
----------------

----------------
X AND Y COORDINA
TES OF C/M
          38.4588
           3.7014

HEIGHT C/M:
          45.7127
----------------

AVERAGE VELOCITY
 OF C/M BETWEEN
FRAME     1.0000
          2.0000
Vh (m/s)=  14.68
Vv (m/s)=   0.05
Vlin(m/s)= 14.68

Angle resultant
           0.21
```

```
RUN #          8

Time/Frame
.0213

++++++++++++++++

++++++++++++++++

INTERNAL ANGLES
    (Deg.)

++++++++++++++++

FRAME #        1

HIP           59
KNEE         124
L.LEG         78

------------

FRAME #        2

HIP           71
KNEE         124
L.LEG         80

------------

++++++++++++++++

   VELOCITIES

++++++++++++++++

F interval
5

Vel. F#        1
               2
m/-          14.54
```

TABLE VI: Symbols and definitions

SYMBOL	DEFINITION	MATHEMATICAL REPRESENTATION	DIMENSION
F_a	Net accelerating force	mg sin A - mg cos A u	N
F_d	Drag force	$\frac{d}{2} V^2$ PA CD	N
F_f	Friction force	mg cos A u	N
F_n	Normal component of the force	mg cos A	N
F_x	Force component parallel to the slope	mg sin A	N
A	Angle of the slope		degrees
A_c	Critical angle for impending motion	$\tan^{-1} u$	degrees
m	Mass of the skier + equipment		Kg
u	Coefficient of friction		
g	Acceleration due to gravity		m/s^2
D	Density of the air		Kg/m^3
PA	Planform area		m^2
CD	Coefficient of drag		

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TABLE VI: Symbols and definitions (cont.)

SYMBOL	DEFINITION	MATHEMATICAL REPRESENTATION	DIMENSION
V	Velocity of the skier	$V^2 = \frac{2mg\ (\sin A - \cos A\ u)}{d\ (PA)\ (CD)} \times \left(1 - e^{\frac{-d\ (PA)\ (CD)\ s}{m}}\right)$	m/s
V_s	Velocity of the skier after displacement s	$V_s = V_t \sqrt{\left(1 - e^{\frac{-d\ (PA)\ (CD)\ s}{m}}\right)}$	m/s
V_t	Terminal velcoity	$V_t = \sqrt{\frac{2mg\ (\sin A - \cos A\ u)}{d\ (PA)\ (CD)}}$	m/s

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APPENDIX C

EQUATIONS

Equation to Calculate the Value of the Critical Angle $A_{(c)}$ for Impending Motion for a Given Coefficient of Friction

A skier will start sliding when the accelerating force is bigger than the friction force. As V = 0 drag need not be considered. Therefore

$$\sin A - \cos A\, u = 0$$

is the condition for impending motion.

Solving for u:

$$\frac{\sin A}{\cos A} = u \; ; \quad u = \tan A$$

The critical value of A for impending motion can be obtained as:

$$A_c = \tan^{-1} u \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Equation for Calculating the Influence of Mass on the Terminal Velocity

For an increasing mass the terminal velocity (V_t) will increase depending on mass 1 (m1) and mass 2 (m2). Recalling the basic (2) equation and allowing for a variable mass, we obtain:

$$\frac{d}{2} V_1^2 \quad PA\ CD = m1\ g \sin A - m1\ g \cos A\ u$$

$$\frac{d}{2} V_2^2 \quad PA\ CD = m2\ g \sin A - m2\ g \cos A\ u$$

Equating both expressions and eliminating the constants:

$$\frac{V_1^2}{m1} = \frac{V_2^2}{m2}$$

Solving for V_2:

$$V_2 = \sqrt{\frac{m2}{m1}} \cdot V_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (7)$$

We define $m2 > m1$; therefore $V_2 > V_1$.

With this equation the increase in V_t due to a mass increment can be calculated.

Example: Assume a skier of mass 75 kg. (with equipment) attaining a V_t of 30 m/s. By increasing his mass to 76 kg. he will increase his V_t to: $V_2 = 30\ (1.0066) = 30.20$ m/s.

Equation to Determine the Influence of the Coefficient of Friction on the Terminal Velocity

For an increasing coefficient of friction the terminal velocity will decrease depending on the two coefficients (u1 and u2) and the angle of the slope.

Recalling the basic (2) equation and allowing for a variable u:

$$\frac{d}{2} V_1^2 \; PA \; CD = mg \sin A - mg \cos A \; u \; 1$$

$$\frac{d}{2} V_2^2 \; PA \; CD = mg \sin A - mg \cos A \; u \; 2$$

Equating both expressions and eliminating the constants:

$$\frac{V_1^2}{\sin A - \cos A \; u \; 1} = \frac{V_2^2}{\sin A - \cos A \; u \; 2}$$

Solving for V_1:

$$V_1 = V_2 \sqrt{\frac{\sin A - \cos A \; u \; 1}{\sin A - \cos A \; u \; 2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

We define u1 > u2; therefore $V_1 < V_2$.

This equation can be used to calculate the loss in V_t when increasing the coefficient of friction.

Example: Assume a skier on a 25° slope attaining a velocity of 26 m/s. with a u of .02. When increasing the coefficient to .05 his new V_t will be: 26 (.9658) = 25.11 m/s.

To obtain the gain in V_t due to a reduction of the coefficient of friction we solve for V_2:

$$V_2 = \frac{V_1}{\sqrt{\frac{\sin A - \cos A\, u\, 1}{\sin A - \cos A\, u\, 2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Example: A skier achieved a V_t of 15 m/s. on a 10° slope. By reducing u from .08 to .06 he will increase his V_t to:

$$\frac{15}{.9097} = 16.49 \text{ m/s.}$$

Equations to Calculate the Influence of the Planform Area and the Coefficient of Drag on the Terminal Velocity

1. Planform Area

For a decreasing planform area (PA) the terminal velocity (V_t) will increase depending on PA1 and PA2.

Recalling the basic (2) equation and allowing for a variable PA we obtain:

$$\frac{d}{2} V_1^2 \, PA1 \quad CD = m\, g \sin A - m\, g \cos A\, u$$

$$\frac{d}{2} V_2^2 \, PA2 \quad CD = m\, g \sin A - m\, g \cos A\, u$$

Equating both expressions and eliminating the constants:

$$V_1^2 \, PA1 = V_2^2 \, PA\, 2$$

Solving for V_1:

$$V_1 = V_2 \sqrt{\frac{PA2}{PA1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

We define PA1 > PA2; therefore $V_1 < V_2$.

This equation can be used to calculate the loss in V_t when the planform area is increased.

Example: A skier reaching a V_t of 29 m/s. with a PA of .6 m^2 will reduce his V_t to: $V_1 = 29\ (.8165) = 23.68$ m/s.

Solving for V_2 we obtain the increased velocity due to a reduction of planform area:

$$V_2 = V_1 \sqrt{\frac{PA1}{PA2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Example: By reducing his planform area from .4 m^2 to .3 m^2 a skier will increase his terminal velocity by 15.47%. Assuming

V_1 = 27 m/s; PA1 = .4 m^2; PA2 = .3 m^2

V_2 = 27 (1.1547) = 31.18 m/s.

2. The Coefficient of Drag

The influence of coefficient of Drag can be determined equivalent to the procedure used to determine the influence of the planform area.

The equations changed to (substituting CD for PA):

$$V_1 = V_2 \sqrt{\frac{CD2}{CD1}} \quad \text{..... (8)}$$

and

$$V_2 = V_1 \sqrt{\frac{CD1}{CD2}} \quad \text{..... (9)}$$

APPENDIX D

COMPUTER PROGRAMS

PROGRAM INDEX

1. Program to compute air resistance.
2. Program to compute terminal velocity.
3. Program to compute per-cent increase in terminal velocity for a given decrease in the coefficient of friction.
5. Program to compute the per-cent change in terminal velocity for a given increase in the coefficient of friction.
6. Program to compute internal body angles and the velocity of a surface mark.
7. Program to compute the velocity of the center of mass of the skier.
8. Program to compute terminal velocity using a formula derived by TEMPLIN (1968).
9. Program to compute time loss due to an increase in planform area for a defined time interval using a formula derived by THOMANN (1976).
10. The velocity of a skier as a function of the displacement.

PROGRAM 1

```
0: fxd 4
1: prt "AIR RESISTANCE";spc 3
2: prt "****************";spc 2
3: 1)N;dim A[1,10];dim B[1,10]
4: ent "Planform Area ?",A
5: ent "Coefficient of Drag ?",L
6: ent "Velocity ?",V
7: ent "V increment?",I
8: prt "Planform",A;prt "Velocity",V;prt "V incr",I
9: prt "C of Drag",L;spc 2
10: for B=1 to 10
11: .6V^2L*A)R
12: R)A[1,N];V)B[1,N]
13: V+I)V;N+1)N
14: next B
15: 1)N;prt "AIR RESISTANCE  (NEWTON)";spc
16: for C=1 to 10
17: prt A[1,N]
18: N+1)N;next C
19: spc ;1)N;prt "VELOCITY        (M/S)";spc
20: for D=1 to 10
21: prt B[1,N]
22: N+1)N;next D;spc 2;prt "****************";spc 2
23: 1)N;gto 4
*26016
```

PROGRAM 2

```
0: prt "TERMINAL          ";prt "VELOCITY";spc 2
1: prt "Downhill-Model";spc 3
2: dim A[10]
3: dsp "TERMINAL VELOCITY";wait 1500
4: ent "Angle of Slope",A
5: ent "Mass of Skier and Gear",M;M)r5
6: ent "Planform Area",C
7: ent "Coefficient of Friction",D
8: ent "Coefficient of Drag",E
9: ent "Mass increment ?",r0
10: ent "Angle increment",r7
11: prt "*-*-*-*-*-*-*-*-"
12: fxd 2
13: spc 2;prt "ANGLE",A;prt "MASS",M;prt "PLANFORM",C;prt "CofFRICTION",I
14: prt "CofDRAG",E;prt "MASS INC.",r0;spc 2;prt "+*+*+*+*+*+*+*+*+";spc
15: 1)N
16: for Z=1 to 10
17: (sin(A)*9.81*M-cos(A)*9.81*M*D)/(.625*E*C))r6
18: if r6<0;spc ;fxd 2;prt "NO MOTION";spc ;gto 31
19: if r6=0;spc ;prt "MOTION IMPENDING";spc ;gto 31
20: fxd 4
21: \((sin(A)*9.81*M-cos(A)*9.81*M*D)/(.625*E*C)))V;V)A[N]
22: fxd 4
23: N+1)N
24: prt "Vt =",V;M+r0)M
25: next Z
26: spc 2
27: 2)N
28: for X=1 to 9
29: A[N]-A[N-1])r1
30: prt r1;N+1)N;next X
31: spc 3;prt "))Angle",A+r7;A+r7)A;spc 2;r5)M;gto 15
*14487
```

PROGRAM 3

```
0: prt "% INCREASE Vt";spc 3
1: 60)M;61)N)K;1)I;1)J;10)A
2: dim A[56,11]
3: for F=1 to 55
4: for A=1 to 10
5: A/\(M/N))B;((B-A)/B)100)C;if C).0001:C)A[I,J]
6: J+1)J;N+1)N
7: next A
8: M+1)M;K+I)N;1)J;I+1)I
9: next F
10: 1)I;1)J;60)X)W
11: for E=1 to 55
12: spc ;fmt 2f4.1;wrt 16,X," Ko";spc
13: for D=1 to 10
14: if A[I,J]<.0001;jmp 2
15: fmt 2f6.3;wrt 16,A[I,J]
16: J+1)J;W+1)W
17: next D;spc 2
18: 1)J;I+1)I;X+1)X)W;next E
19: end
*21907
```

PROGRAM 4

```
0: prt "& INCREASE Vt BY      CHANGING   ←→FRICTION";spc 3
1: enp "      ANGLE",A;.1→B;.01→C;10→V;spc 2;fxd 3
2: for J=1 to 10
3: for I=1 to 10
4: V/((sin(A)-cos(A)B)/(sin(A)-cos(A)C))→R
5: ((V-R)/V)100→D;abs(D)→D
6: prt char(12),B,char(12),C;prt "&inc.",D;spc
7: C-.01→C
8: if C=0;jmp 2
9: next I
10: B-.01→B;B-.01→C;spc 3
11: if B=.01;prt "DONE";spc 2;end
12: next J
13: end
*RUN
```

PROGRAM 5

```
0: prt "Decreasing VT for increasing coefficient of friction";spc 3
1: enp "ANGLE",A;.1→C;.09→B;spc 2
2: for J=1 to 10
3: for I=1 to 10
4: ((sin(A)-cos(A)C)/(sin(A)-cos(A)B))→R
5: prt char(12),B,char(12),C;prt "Factor",R;spc
6: B-.01→B
7: if B=0;jmp 2
8: next I
9: C-.01→C;C-.01→B;spc 3
10: if C=.01;prt "DONE";spc 3;end
11: next J
12: end
*9797
```

PROGRAM 6

```
0: prt "INTERNAL ANGLES"
1: prt "VELOCITY (Hip)";spc 3
2: dim Z$[4,25];1)Q
3: ent "NUMBER OF FRAMES ?",R
4: dim Z[2,R]
5: dsp "SET ORIGINE ON X - AXIS";wait 2000
6: dsp "DIGITIZE SECOND POINT ON X-AXIS";red 4,A,B
7: \(A^2+B^2))H
8: B/H)K;A/H)L
9: dsp "X-AXIS ROTATED TO",asn(K),"DEGREES";wait 1500
10: prt "X-ROTATION",asn(K);spc
11: enp "CORRECTIONFACTOR",r17;spc
12: prt "----------------";spc
13: ent "Number of runs",P;1)r22
14: for W=1 to P
15: spc ;prt "*****************";spc
16: fxd 0;prt "RUN #",r22;r22+1)r22;1)Q;spc ;fxd 2
17: enp "Time/Frame",r20;spc
18: spc ;prt "*****************";spc
19: prt "+++++++++++++++";spc
20: prt "INTERNAL ANGLES";prt "      (Deg.)";spc
21: prt "+++++++++++++++++";spc
22: for M=1 to R
23: fxd 0;prt "FRAME #",Q;fxd 2;spc
24: "ACROMION (Shoulder)")Z$[1]
25: "G. TROCHANTER (Hip)")Z$[2]
26: "LAT. CONDYLE (Knee)")Z$[3]
27: "LAT. MALLEOLUS (Ankle)")Z$[4]
28: dsp Z$[1];red 4,A,B;beep
29: dsp Z$[2];red 4,C,D;beep
30: dsp Z$[3];red 4,E,F;beep;2.54E)Z[1,Q];2.54F)Z[2,Q]
31: dsp Z$[4];red 4,G,H;beep
32: AL+BK)r5;BL-AK)r6;CL+DK)r7;DL-CK)r8
33: EL+FK)r9;FL-EK)r10;GL+HK)r11;HL-GK)r12
34: r5)A;r6)B;r7)C;r8)D;r9)E;r10)F;r11)G;r12)H
35: \((A-C)^2+(B-D)^2))r0
36: \((A-E)^2+(B-F)^2))r1
37: \((C-E)^2+(D-F)^2))r2
38: acs((r0^2+r2^2-r1^2)/2r2r0))Z
39: \((C-E)^2+(D-F)^2))r0
40: \((E-G)^2+(F-H)^2))r2
41: \((C-G)^2+(D-H)^2))r1
42: acs((r0^2+r2^2-r1^2)/2r2r0))Y
43: prt "HIP",Z;prt "KNEE",Y
44: atn((F-H)/(E-G)))X;prt "L.LEG",X;spc ;prt "  ------------  ";spc
45: Q+1)Q;next M
46: prt "+++++++++++++++++";spc
47: prt "   VELOCITIES";spc ;prt "+++++++++++++++++";spc
48: enp "F interval",r30;spc
49: 1)Q
50: for O=1 to R-1
51: \((Z[1,Q+1]-Z[1,Q])^2+(Z[2,Q+1]-Z[2,Q])^2)r17)r18;r18/100)r18
52: fxd 0;prt "Vel. F#",Q,Q+1;fxd 2;prt "m/s",r18/r30r20;spc 2;Q+1)Q
53: next O
54: next W
55: end
*12592
```

PROGRAM 7

```
0: prt "ANALYSIS OF DOWNHILL SKIING";spc 3
1: 1→N;1→I;1→J;0→F;0→U;0→W;0→r0;1→r1;0→r2;1→Z;1→r12
2: dim A$[50]
3: ent "COMMENTS ( up to 50 Characters )",A$[N];if N=16;beep
4: prt "*****************";spc
5: prt A$[N];spc
6: prt "*****************";spc 3;fxd 0
7: enp "NUMBER OF RUNS",r7
8: enp "NUMBER OF FRAMES",N;fxd 2
9: enp "C FACTOR",r11
10: dsp "       RESET ORIGINE";wait 2000
11: dsp "INPUT SECOND POINT ON X AXIS"
12: red 4,A,B;√(A^2+B^2)→H
13: B/H→r25;A/H→r26
14: dsp "X AXIS ROTATED TO",asn(r25),"DEGREES";wait 2000
15: spc ;prt "X-ANGLE",asn(r25);spc 2
16: dim X[2,7]
17: dim M[1,14]
18: dim A[1,14]
19: dim Z$[14,11]
20: .5→A[1,1]
21: .5→A[1,2]
22: .436→A[1,3]
23: .43→A[1,4]
24: .23→A[1,5]
25: .436→A[1,6]
26: .43→A[1,7]
27: .23→A[1,8]
28: .433→A[1,9]
29: .433→A[1,10]
30: .45→A[1,11]
31: .433→A[1,12]
32: .433→A[1,13]
33: .45→A[1,14]
34: .096→I[1,1]
35: .453→I[1,2]
36: .033→I[1,3]
37: .019→I[1,4]
38: .0055→I[1,5]
39: .033→I[1,6]
40: .019→I[1,7]
41: .0055→I[1,8]
42: .105→I[1,9]
43: .045→I[1,10]
44: .0145→I[1,11]
45: .105→I[1,12]
46: .045→I[1,13]
47: .0145→I[1,14]
48: "Head & Neck"→Z$[1]
49: "Trunk"→Z$[2]
50: "R.Upper Arm"→Z$[3]
51: "R.Lower Arm"→Z$[4]
52: "R.Hand"→Z$[5]
53: "L.Upper Arm"→Z$[6]
54: "L.Lower Arm"→Z$[7]
55: "L.Hand"→Z$[8]
56: "R.Thigh"→Z$[9]
57: "R.Lower Leg"→Z$[10]
58: "R.Foot"→Z$[11]
59: "L.Thigh"→Z$[12]
```

PROGRAM 7 (cont'd)

```
49: "R.Thigh "&Z$[12]
50: "L.Lower Leg"&Z$[13]
51: "L.Foot"&Z$[14]
52: for G=1 to r7
53: fxd 0;prt "******************";spc
54: prt "RUN #",G;spc ;fxd 4
55: enp "TIME/FRAME",r3;enp "# FRAMES",r3;r3r8+r3;spc
56: for H=1 to N
57: 1+N;1+M;1+J;0+T;0+U
58: for I=1 to 14
59: dsp "PROXIMAL",Z$[N];red 4,A,B;wait 500;be
60: dsp "DISTAL",Z$[N];red 4,X,Y;beep
61: A*2.54+A;X*2.54+X;B*2.54+B;Y*2.54+Y
62: Yr26+Br25+r27;Br25-Ar25+r28
63: Xr25+Yr25+r29;Yr25-Xr25+r30
64: r27+V;r28+B;r29+X;r30+Y
65: A-X+D;abs(D)+O
66: B-Y+P;abs(P)+P
67: O*A[1,M]+K;P*A[1,M]+L
68: if A<X;K+A+E
69: if A>X;A-K+E
70: if B<Y;L+B+F
71: if B>Y;B-L+F
72: fxd 4
73: M[1,J]+V
74: E*V+C
75: F*V+D
76: C+T+R;D+U+Q
77: N+1+N;M+1+M;J+1+J;R+T;Q+U
78: next I
79: ent "ERROR ? (0=YES)",S
80: if S=0;dsp "DIGITIZE FRAME AGAIN";wait 2000;prt "ERROR*******";gto 67
81: prt "-----------------"
82: prt "X AND Y COORDINATES OF C/M",R,Q;spc
83: prt "HEIGHT C/M:",Qr11
84: prt "-----------------";spc
85: R+X[1,Z]
86: Q+X[2,Z]
87: Z+1+Z
88: r0+1+r0
89: next d
90: 1+Z
91: for S=1 to N-1
92: Z+1+r1
93: X[1,Z]-X[1,r1]+r13
94: X[2,Z]-X[2,r1]+r14
95: √((X[1,r1]-X[1,Z])^2+(X[2,r1]-X[2,Z])^2)+r2
96: r1+Z
97: fxd 4
98: r13*r11/100/r3+r15
99: r14*r11/100/r3+r16
100: r2*r11/100+r2;r2/r3+r2
101: if r12>1;gto 114
102: r12+1+r12
103: fxd 0
104: prt "AVERAGE VELOCITY OF C/M BETWEEN FRAME",Z-1,Z
105: fxd 2
106: prt "Vh (m/s)=",r15(-1);prt "Vv (m/s)=",r16(-1);prt "Vlin(m/s)=",r2;spc 1
107: r14/r13+r17;deg;atn(r17)+r17
108: prt "Angle resultant",r17;spc 5
109: next G
110: next G
111: end
*27515
```

PROGRAM 8

```
0: dim A[10]
1: prt "TERMINAL"
2: prt "VELOCITY":spc 3
3: ent "Lenght of run ?",A
4: ent "Height difference ?",B
5: ent "Coefficient of Friction ?",C
6: ent "Mass of skier + gear",D:D*9.8
7: ent "mass increment ?",I:I*9.81)I
8: ent "Planform Area ?",E
9: ent "Coefficient of Drag ?",F
10: 1)N
11: for H=1 to 10
12: \(2(B/A-C)*D/(1.2*(E*F)))\V:V)A[N]
13: asn(B/A))G
14: fxd 2
15: prt "mass",D/9.81
16: fxd 4;prt "V max (m/s)",V;spc
17: D+I)D:N+1)N
18: next H
19: prt "----------------"
20: prt "Angle",G
21: prt "Fk",C
22: prt "Planform",E
23: prt "C of Drag",F
24: spc :prt "******************";spc 2
25: ent "Output Vx-Vx-1 ? (0=Yes)",K
26: if K=0
27: 1)N
28: for X=1 to 9
29: A[N+1]-A[N])Z
30: prt Z
31: N+1)N
32: next X
*4401
```

PROGRAM 9

```
0: prt "TIME LOSS DUE TO INCREASED PLAN-FORM AREA / TIME";sdc 2
1: prt "-------------------"
2: sdc
3: ent "Original Planform Area",A
4: ent "New Planform Area",B
5: ent "TIME INTERVAL",C
6: (C/2)((B-A)/A))D
7: (B-A)/A)E;E*100)E
8: prt "Old Planform",A
9: prt "New Planform",B
10: prt "Time interval",C;sdc
11: prt "% INCREASE OF PLANFORM AREA",E
12: prt "PERMANENT TIME  LOSS (sec)",D;sdc :gto 1
*
```

2065

PROGRAM 10

```
0: prt "VEL.ofSKIER(m/s)";spc 2
1: ent "Mass",M;ent "Angle",A
2: ent "CofFric",F;ent "Planform",P
3: ent "CofDrag",C
4: if sin(A)-cos(A)F<=0;dsp "N O  M O T I O N";wait 2000;gto 1
5: prt "Mass",M;prt "Angle",A;prt "CofFric",F
6: prt "Planform",P;prt "CofDrag",C
7: √((sin(A)-cos(A)F)19.62M/1.25PC)→T
8: spc ;prt "TERMINAL VEL.",T;spc
9: gsb "dash"
10: prt "  DISP.      VEL.";gsb "dash";spc
11: for Z=1 to 51
12: √(1-exp(-1.25PCS/M))T→V
13: fmt 2f8.2;wrt 16,S,V
14: S+10→S;next Z
15: spc 2;end
16: "dash":
17: prt "----------------"
18: ret
*19420
```